VALIDATION OF THE NUMERICAL SIMULATION OF THE SCATTERED SOUND FIELD WITH A GEOMETRICAL PYRAMID TRACING APPROACH

A. Farina Industrial Engineering Dept., Università di Parma, Via delle Scienze 181/A
43100 PARMA - Italy - tel. +39 0521 905854 - fax +39 0521 905705
E-MAIL: farina@pcfarina.eng.unipr.it – HTTP://pcfarina.eng.unipr.it

1. INTRODUCTION

The paper describes the details and some validation results about a new implementation of sound scattering modelling inside a Pyramid Tracing computer program, for the simulation of the sound propagation in large rooms and outdoors. The approach makes it possible to take into account both scattering mechanisms (surface and edge), along with the frequency dependence of both effects.
The surface scattering is described by means of the scattering coefficient (a physical quantity actually being standardised by an ISO WG) in each octave band; the edge scattering, instead, is computed empirically with reference to an hypothetical flat and smooth surface, and this effect is added to the surface scattering.
A clever implementation made it possible to maintain an unique geometrical tracing for all the frequency bands, and thus the computation time is not increased over the previous version of the same program, which did not include explicitly the scattering effect.
The comparison with experimental measurements and theoretical solutions has shown that the new algorithm greatly reduces the computational error in presence of strong reflections from scattering objects, provided that the correct value of the scattering coefficient is employed as input.

2. BACKGROUND

Geometrical room acoustics programs revealed soon that, in practical usage, the hypothesis of specular reflection does not hold. Consequently, most modern programs based on hybrid approach are employing some sort of simulation of the scattering phenomena [1,2,3]. In some cases, the implementation of the scattering effects required very complex approaches [4], and usually the frequency dependence of the scattering requires to repeat the geometrical tracing for each frequency band, causing intolerable computation times.
In most cases, these programs require that user assigns a proper value of the scattering coefficient, for each frequency band and for each surface. The scattering coefficient δ is defined as the ratio between the reflected energy spread around in a diffuse way and the total reflected energy:

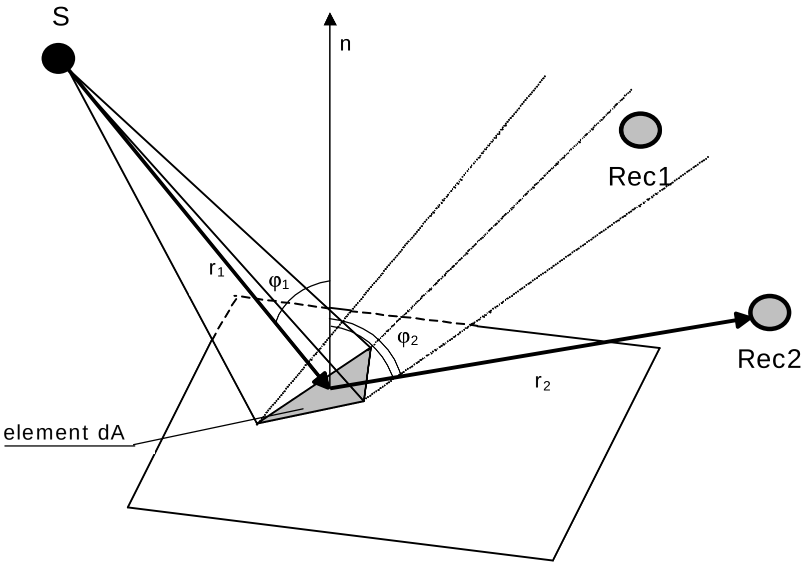
$$\delta = \frac{E_{diff}}{E_{diff} + E_{spec}}$$

It is not clear how the reflected energy can be separated into specular and diffuse components, although some experimental methods were proposed [5]. Another approach, more pragmatic, is simply to find the “optimal” value of δ for each case, which is the value that minimizes the deviation between the numerical results and the experimental data [6].
In this scenario, the pyramid tracing algorithm (which is not hybrid) continued to be employed without any explicit consideration for surface or edge scattering effects [7,8,9]. It was surprising how this algorithm appeared to be more immune from the artefacts which afflicted other competing algorithms not taking into account the scattering phenomena: only very recently it was discovered that the original pyramid tracing scheme already contained a sort of intrinsic diffusion of the late energy [10], and this explained the reasonably good performances obtained with very little number of pyramids. Furthermore, this explained also why, increasing the number of pyramids, instead of

obtaining an amelioration of the overall accuracy, the program seemed to deviate systematically from the experimental results: the statistical behaviour was being pushed towards longer times, and the specular behaviour was being extended to higher order reflections.
The author did not attempted to include an explicit treatment of scattering surfaces because, until a practical method for measuring experimentally the scattering coefficient was available, there was no point in asking to the user to guess for reasonable values of it, with the risk of causing errors much greater than those produced by the original algorithm with little number of pyramids (and consequently very short computation times).
But now both AES and ISO are preparing new standards for the experimental characterisation of the scattering properties of materials, and the new method proposed in [6] seems capable of producing results compatible with both standards requiring only a minimum effort. Thus, it is time to modify the original pyramid tracing, introducing the capability of modelling explicitly the frequency-dependent scattering coefficient of the surfaces, and the scattering caused by the edges of each panel.
For continuity with the previous implementation, anyway, the constraints of maintaining the algorithm not hybrid, completely deterministic, and very fast were posed. In practice, only a minimal modification had to be done to the computing formulas, without the need of a separate geometrical tracing for each frequency band.
In the following, first the theoretical formulation is given, then this is compared with the previous implementation. Two kinds of verification were finally attempted: comparison with experimental results in the case of first-order reflection on an highly diffusing surface, and comparison with statistical theoretical formulations in the case of a diffuse-field reverberating room.

3. THEORY

The algorithm described here is an extension of the pyramid tracing computer model Ramsete [7,8,9]. The pyramid tracing algorithm is well established nowadays, so it will not described completely here.
The following picture shows what happens when a pyramid is reflected over a surface, with reference to two receivers, one which happens to be within the prosecution of the currently traced pyramid, the other outside of it.



In the original formulation (no scattering), only the first receiver receives an amount of acoustic intensity, computed following this equation:

$$I_{orig} = \frac{W \cdot Q_{\vartheta}}{4 \cdot \pi \cdot \left(r_{1,tot} + r_2\right)^2} \cdot \prod_{i=1}^{N} \left(1 - \alpha_i\right) \cdot e^{-\gamma \cdot \left(r_{1,tot} + r_2\right)}$$

in which W is the acoustic power of the source, Q_{ϑ} is the directivity coefficient in the direction of original emission of the pyramid, $r_{1,tot}$ is the total path of the ray until the last reflection point, r_2 is the distance from the last reflection point and the receiver, α_i are the absorption coefficients of the surfaces where the pyramid was reflected and γ is the air absorption coefficient.

Instead, when scattering is taken into account, the receiver n. 1 gets two different contributions (specular and diffuse), whilst receiver n. 2 gets only the latter. The specular contribution is computed as follows:

$$I_{spec} = \frac{W \cdot Q_{\vartheta}}{4 \cdot \pi \cdot \left(r_{1tot} + r_2\right)^2} \cdot \left[\prod_{i=1}^{N} \left(1 - \alpha_i\right)\right] \cdot (1 - \delta_{tot}) \cdot e^{-\gamma \cdot \left(r_{1,tot} + r_2\right)}$$

The equation for the specular intensity is easy to understand, as it differs from the original formulation only for the fact that the factor (1- δ_{tot}) was included. This means that the value of δ_{tot} separates the whole energy transported by the pyramid in a diffuse part and in a specular part.

δ_{tot} depends on the values of the scattering coefficients of the surfaces where the pyramid was reflected (it is a sort of "cumulative scattering coefficient"): it always starts with an initial value of zero (direct sound), and reflection after reflection its value increases, approaching 1. After the N-th reflection, its value is obtained from the previous (N-1)th value and the local value of the scattering coefficient applicable for the point where the pyramid axis was reflected:

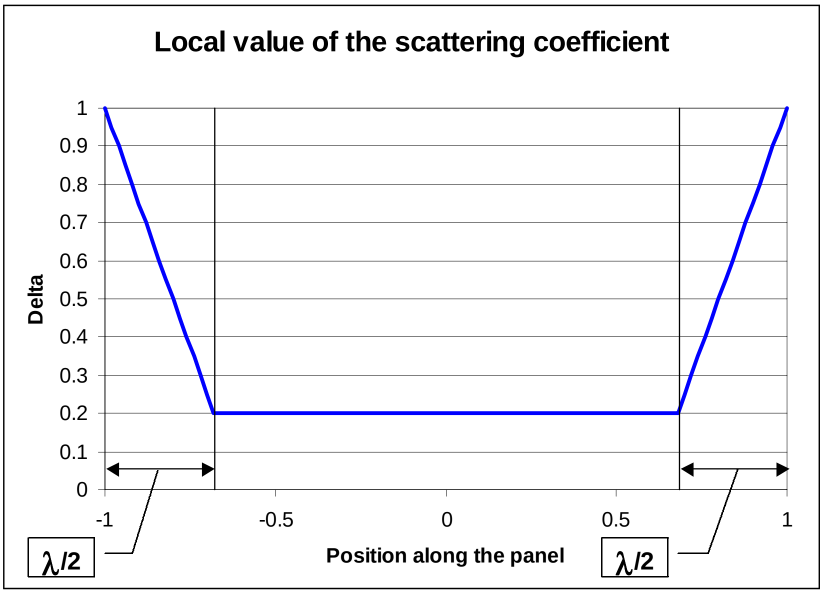
$$\delta_{tot,N} = \delta_{tot,N-1} + \left(1 - \delta_{tot,N-1}\right) \cdot \delta_{loc}$$

Also in this case the meaning is easy to understand: at each reflection, the energy which is already diffuse remains diffuse, but a fraction of the specular energy proportional to δ_{loc} commutes to diffuse, so that the diffuse part always increases.

The local value of the scattering coefficient δ_{loc} depends on the scattering properties of the material assigned to the reflecting surfaces, described by the frequency-dependent scattering coefficient δ, and on the distance from the nearest edge of the surface. The scattering coefficient δ is defined as the ratio between the energy reflected in a diffuse way and the total reflected energy, for an infinite surface; for finite-size surfaces, the local value increases near the edges. A very simplified formulation of this phenomenon is considered inside the numerical model described here, based on a linear interpolation between the infinite surface coefficient δ, at a distance from the edge greater or equal to half the wavelength λ, and the value 1, when the reflection point is on the edge of the panel. Denoting the distance of the reflection point from the nearest edge with letter s, it becomes:

$$\delta_{loc} = 1 - (1 - \delta) \cdot \frac{s}{\lambda / 2}$$

The following picture shows the spatial distribution of δ_{loc} along a panel.



Finally, we have to derive a consistent formulation for the diffuse intensity. The first step is to compute the total power reflected from the triangular area dA obtained by the intersection of the pyramid and the reflecting surface:

$$W_{refl} = \frac{W \cdot Q_\vartheta}{N_{pyr}} \cdot \prod_{i=1}^{N} (1-\alpha_i) \cdot e^{-\gamma \cdot r_{1,tot}}$$

The fraction δ_{tot} of this power is being spread uniformly in the hemispace in front of the reflecting surface, following the uniform intensity model (constant directivity of the diffuse intensity, not the cosine-varying Lambert's model). Thus, for a receiver located at the distance r_2 from the centre of the elementary reflecting area dA, the diffuse intensity should be:

$$I_{diff} = \frac{W \cdot Q_\vartheta \cdot \left[\prod_{i=1}^{N} (1-\alpha_i)\right] \cdot \delta_{tot}}{N_{pyr} \cdot 2 \cdot \pi \cdot r_2^{\,2}} \cdot e^{-\gamma \cdot (r_{1,tot} + r_2)}$$

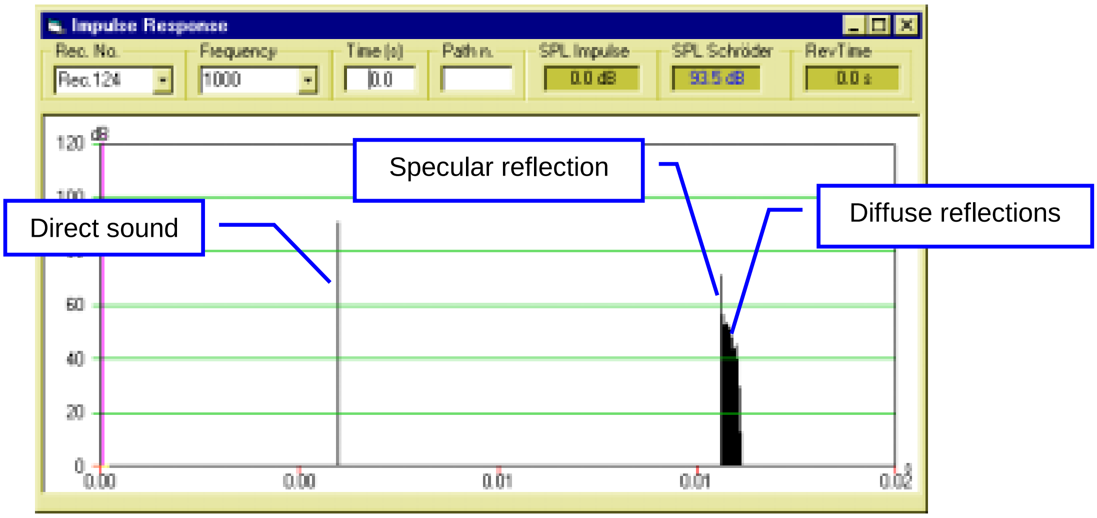
This is fine for receivers not too close to the reflection point. But, if r_2 is made very small, it can happen that the diffuse intensity computed with the above equation becomes terribly high, even larger than the total diffuse intensity, which was impinging over the area dA. This means that, when r_2 is small, the diffuse power has to be diluted, at least, on the elementary area $dA = 4 \cdot \pi \cdot r_{1,tot}^{\,2} / N_{pyr}$:

$$I_{diff,max} = \frac{W \cdot Q_\vartheta \cdot \left[\prod_{i=1}^{N} (1-\alpha_i)\right] \cdot \delta_{tot}}{4 \cdot \pi \cdot r_{1tot}^{\,2}} \cdot e^{-\gamma \cdot r_{1,tot}}$$

Finally, considering simultaneously the dilution over those two surfaces, as described by the previous two equations, we obtain the comprehensive expression of the diffuse intensity:

$$I_{diff} = \frac{W \cdot Q_\vartheta \cdot \left[\prod_{i=1}^{N} (1-\alpha_i)\right] \cdot \delta_{tot}}{4 \cdot \pi \cdot r_{1tot}^{\,2} + N_{pyr} \cdot 2 \cdot \pi \cdot r_2^{\,2}} \cdot e^{-\gamma \cdot (r_{1,tot} + r_2)}$$

In practice, after each reflection, the receivers are checked for being included in the pyramidal beam. If this is true, they receive the specular intensity. Then they are checked for being in the positive hemispace in front of the reflecting surface: if this is true, they receive the diffuse intensity. Thus a receiver, which is inside the pyramidal beam, receives both contributions.
If a large surface is being hit by a large number of narrow pyramids, a given receiver will be inside the reflected beam of only one pyramid, but it will receive diffuse energy from the reflection points of all the pyramids that are hitting on the surface. This means that the specular reflection will produce a sharp peak on the impulse response, followed by a weaker diffuse tail coming from all the other diffuse contributions, as shown in the following picture:



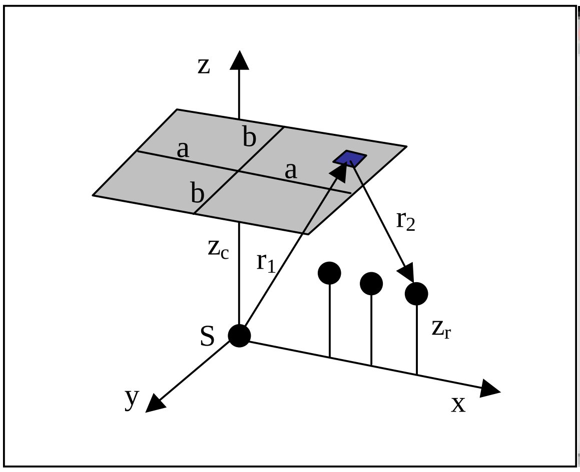
The above described approach does not require that a separate geometrical tracing is done for each frequency band, and thus preserves the very little computation time which was typical of the original pyramid tracing formulation. On the other hand, as demonstrated by the picture above, the new formulation produces realistic impulse responses, where also the first-order reflections are always followed by a diffuse tail, as it always happen with measured impulse responses [11].
But what does happen for the late reverberant tail? When the total path of the pyramid is very long, the term $4 \cdot r_{1,tot}^2$ is much greater than $2 \cdot N_{pyr} \cdot r_2^2$. Furthermore, the pyramidal beam is so huge that all the receivers are always within it, so that they always receive both specular and diffuse intensity. Thus they receive a total intensity given by the sum of diffuse and reflected intensities:

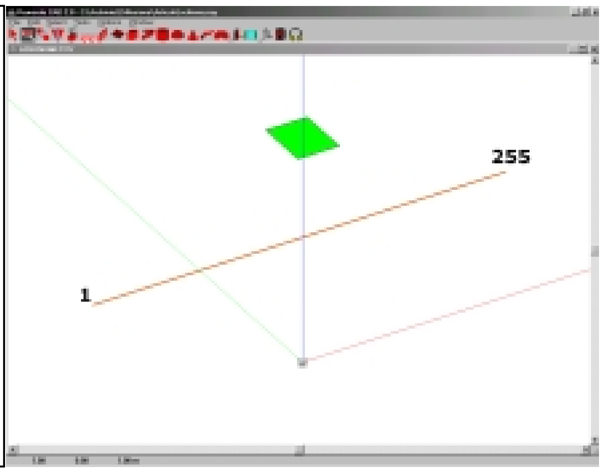
$$I_{tot} = W \cdot Q_{\vartheta} \cdot \left[\prod_{i=1}^{N} (1-\alpha_i) \right] \cdot e^{-\gamma \cdot (r_{1,tot} + r_2)} \cdot \left[\frac{\delta_{tot}}{4 \cdot \pi \cdot r_{1tot}^2 + N_{pyr} \cdot 2 \cdot \pi \cdot r_2^2} + \frac{1-\delta_{tot}}{4 \cdot \pi \cdot (r_{1tot} + r_2)^2} \right]$$

which is substantially equal to the intensity computed with the original algorithm (no scattering). This means that there is no difference in the late reverberant tail, and that the original algorithm intrinsically produced a diffuse reverberant tail.

4. EXPERIMENTAL VERIFICATION

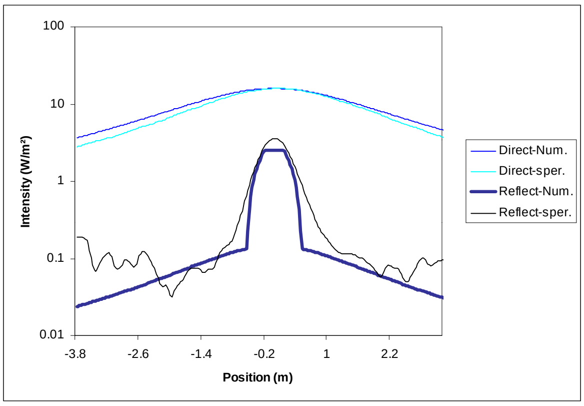
The experimental assessment of the new algorithm was first undertaken for the simulation of the sound field reflected by a diffusing panel, considering only the first-order reflection. The test case was actually realized in the laboratory, suspending a diffusing panel just above a loudspeaker, in an anechoic environment. A movable microphone was employed for measuring the impulse response in 255 positions along a straight line. The following pictures show the schematic apparatus, and the Ramsete CAD model employed for the numerical simulations:



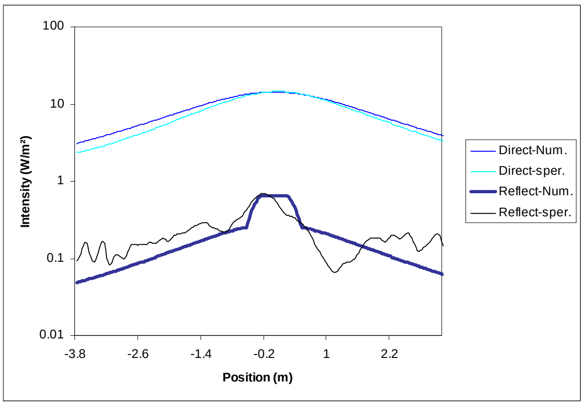


The experiment was repeated with 4 different diffusing panels; details on the experimental apparatus are found in [6]. In each case, the intensities of the direct and reflect sound were compared for the 255 microphones, for evaluating the spatial consistence between numerical simulation and experimental values. The following 4 pictures show the result of the comparison.

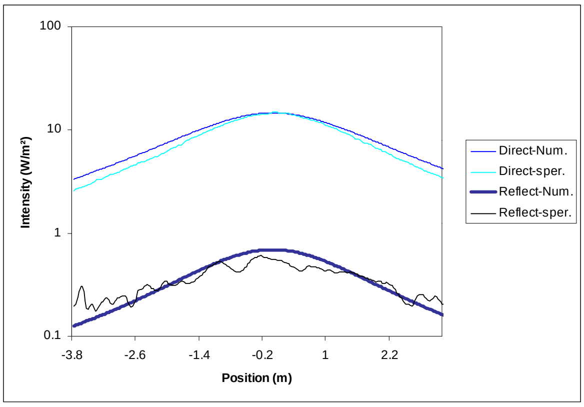
Flat Panel



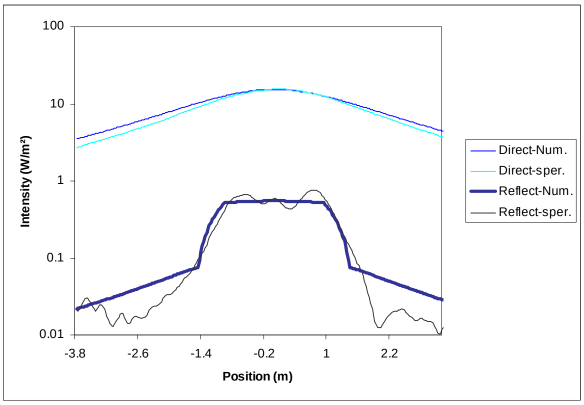
Schroeder-type diffuser



Curved Panel , normal to cylinder axis



Curved Panel , along cylinder axis

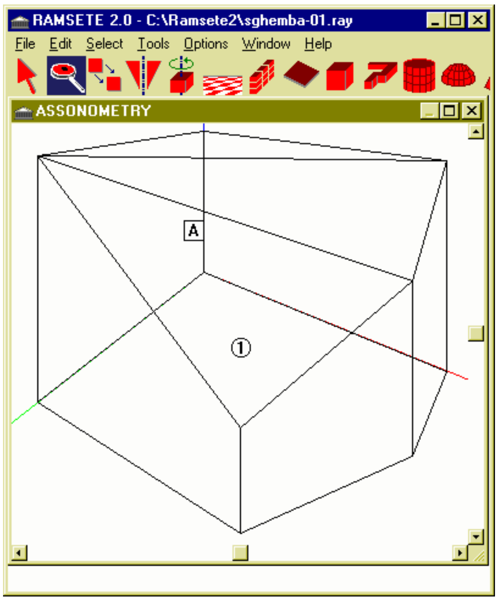


These very satisfactory results were obtained inserting in the computer model the values of the absorption coefficient and of the scattering coefficient measured with the new technique described in [6] – employing wrong values for the input data, of course, cause much wider errors.

5. THEORETICAL VERIFICATION

The second verification was done by verifying that the numerical solution of a diffuse-field reverberant room produces results in close agreement with the theoretical formulations commonly accepted in statistical acoustics, such as the Sabine or Eyring formulas.

The test case was an irregularly-shaped large room, with a little value of the absorption coefficient (0.1 everywhere). The simulation was first conducted with the original formulation of the pyramid tracing algorithm (no explicit scattering) and then with the new one: in the second case, two very different values of the scattering coefficient were employed (0.1 and 0.9). The room volume was 2496 m^3, and the total surface was 1068 m^2. The omnidirectional source was located at 10m from the receiver. The picture on the left shows the geometry of the irregularly-shaped room.

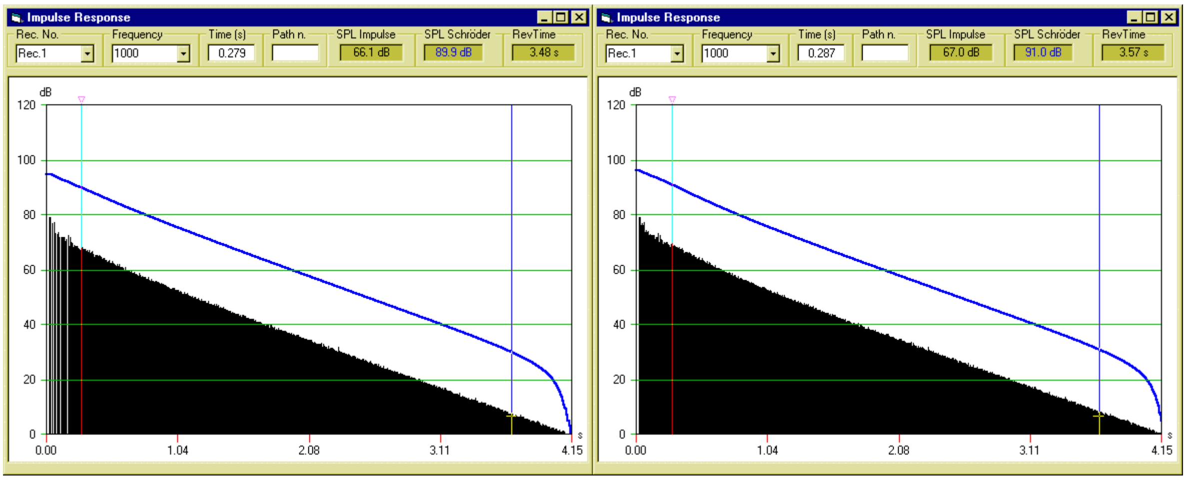


Due to the shape of the room, in principle the values of the sound pressure level and of the reverberation time should not change with the scattering coefficient. The following table compare the theoretical and numerical results:

Case	SPL (dB)	T_{60} (s)
Theoretical (Sabine)	95.7	3.74
Numerical – no scattering	94.9	3.48
Numerical – d = 0.1	96.0	3.54
Numerical – d = 0.9	96.6	3.57

It can be seen that the new formulation reduces the deviation from the theoretical values, but the original formulation was indeed quite good. This confirms that, even without an explicit treatment of the diffuse sound field, the original pyramid tracing algorithm was capable of correctly modelling the statistical behaviour of the late part of the reverberant tail, provided that an optimal number of pyramids (usually quite little) is employed. Thus, the main advantage of the new formulation is to remove the dependence of the accuracy on the optimal choice of the number of pyramids, but this has to by paid by the fact that now it is necessary to input a new set of data: the scattering coefficient value at each frequency band.

The following pictures compares the impulse responses computed with the original algorithm (left) and with the novel one (right), employing 2048 pyramids.



Only minor differences appear in the first 200ms, barely noticeable in the above picture: some gaps, which were present between the first order reflections, are now filled up with diffuse energy.

6. CONCLUSION

A modified pyramid tracing algorithm has been described, which takes explicitly into account the scattering properties of materials. The new formulation does not cause a significant increase in the computation time, nor it requires larger number of pyramids to be launched: thus the algorithm confirms to be one of the faster methods for room acoustics simulation.
The results revealed to be much more adherent to the reality for modelling the first reflections, and tend asymptotically to converge with the results of the original pyramid tracing formulation in the late reverberant tail.
The knowledge of the exact values of the scattering coefficient revealed to be critical for the correct simulation of the first order reflections; instead, for the estimate of robust parameters (such as sound level and reverberation times), which are dominated by the late reverberant tail, the choice of the scattering coefficient values revealed to be much less critical.
In any case, the numerical formulation of the algorithm is perfectly consistent with the measurement technique described in [6], and this suggests that it will be possible to create in a little time a first data base of experimentally measured scattering coefficient values.
The natural prosecution of this work will be the subjective evaluation of the audible simulations obtained by the auralization technique, based on the numerically predicted impulse responses, in comparison with experimental ones.

7. BIBLIOGRAPHY

[1] Naylor G.M. - "ODEON - Another Hybrid Room Acoustical Model" - Applied Acoustics Vol. 38 no.s 2-4, pag. 131 (1993).

[2] B.I. Dalenbäck, "Room acoustic prediction and auralization based on a unified treatment of diffuse and specular reflection" – 15th Intern. Congress on Acoustics, Trondheim, Norway, pp. 425-428, June 1995.

[3] J.J. Embrechts, "Modelization of Kirchoff scattering by a sound ray algorithm" – 16th Intern. Congress on Acoustics, Seattle, USA, vol. 4, pp. 2329-2330, 1998.

[4] O. Warusfel – "Predictive acoustics software and computer aided optimisation in room acoustics" – 15th Intern. Congress on Acoustics, Trondheim, Norway, pp. 693-696, June 1995.

[5] E. Mommertz, M. Vorlander, "Measurement of scattering coefficients of surfaces in the reverberation chamber and in the free field" – Proc. 15th Internat. Congress on Acoustics, Trondheim, 1995, pp. 577-590.

[6] A. Farina, M. Zanolin, E.Crema – "Measurement of sound scattering properties of diffusing panels through the Wave Field Synthesis approach", Pre-prints of the 108th AES Convention, Paris, 19-22 February 2000.

[7] A. Farina - "RAMSETE - a new Pyramid Tracer for medium and large scale acoustic problems" - Euro-Noise 95 Conference, Lyon 21-23 march 1995.

[8] A. Farina - "Pyramid Tracing vs. Ray Tracing for the simulation of sound propagation in large rooms" - In the volume "Computational Acoustics and its Environmental Applications", pp. 109-116, Editor C.A. Brebbia, Computational Mechanics Publications, Southampton (GB) 1995.

[9] A. Farina - "Verification of the accuracy of the Pyramid Tracing algorithm by comparison with experimental measurements by objective parameters" - ICA95 (International Conference on Acoustics), Trondheim (Norway) 26-30 June 1995.

[10] A. Farina, "Introducing the surface diffusion and edge scattering in a pyramid-tracing numerical model for room acoustics", Pre-prints of the 108th AES Convention, Paris, 19-22 February 2000.

[11] R.Torres and M. Kleiner, "Considerations for Including Surface Scattering in Auralization" – Paper 4AAAA-5 - Forum Acusticum, Berlin, march 14-19 1999.