TRANSFER FUNCTION METHOD FOR INVESTIGATING THE DYNAMIC TRANSFER STIFFNESS OF SPRING-LIKE INSULATORS

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1. INTRODUCTION

In this brief paper, an application of the well-known Transfer Function Method (TFM) to the determination of the Dynamic Transfer Stiffness of spring insulators is shown. This parameter is appropriate both to characterize an insulator for vibration control purposes (at frequencies below 100 Hz), both for acoustical transfer (at frequencies up to 2 kHz), as clearly stated in the new test codes ISO/CD 10846 [1].

TFM was originarly developed as a general method to obtain informations about the Complex Modulus of rubber-like materials (Pritz [2]). It has the advantage that measurements can be made with low cost testing equipment, compared to the "direct methods" that require expensive test rigs for dynamic tests.

In the following the original theory is recalled, and then the application to the determination of Dynamic Transfer Stiffness in both vertical and horizontal directions for a rubber cone-type insulator is shown.

2. THEORY

Let us consider a spring-mass system, that is excited by basement vertical motion:

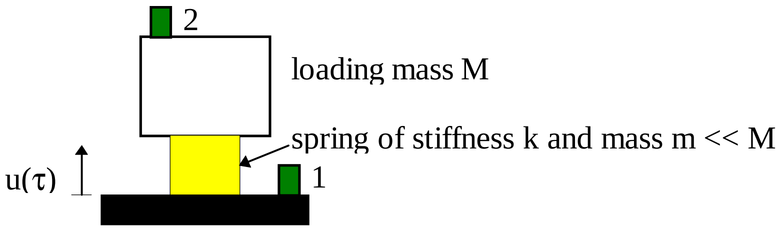

Fig. 1 - scheme of the Transfer Function Method.

Suppose that the excitation velocity u(τ) is obtained with a random noise generator (for example an electrodynamic shaker) or equivalent. Futhermore, two accelerometers are connected to the base moving plate and to the loading mass(1 and 2 in the above picture). Let we define the system Transmissibility T(ω) as the (complex) ratio between the Fourier spectra of the signals obtained from these two accelerometers:

$$T(\omega) = \frac{X_2(\omega)}{X_1(\omega)} = |T| \cdot e^{i\varphi}$$

The TFM theory, developed by Pritz, give us this equation that express the dynamic stiffness k from the measured value of T:

$$k = \omega^2 \cdot M \cdot \frac{|T| \cdot \cos\varphi - |T|^2}{2 \cdot |T| \cdot \cos\varphi - |T|^2 - 1}$$

However, also k must be regarded as a complex number, the expression above giving only the real part of such a complex number. The ratio of the immaginary to real part of k is called the loss factor η:

$$\eta = \frac{\mathrm{Im}[k]}{\mathrm{Re}[k]} = \frac{\sin\varphi}{\cos\varphi - |T|}$$

The two equation above are all that is needed to obtain the (frequency dependent) value of k and η from a single FFT transfer function measurement.

It is important to note that, when the spring mass m is not negligible compared to the load mass M, the previous formulae require only a slight modification: the real part of k is obtained in this case simply multiplicating the expression above for (1+m/2M).

As $|T|$ tends to zero for high frequencies ($\omega \to \infty$) one finds in this case a well known result:

$$\lim_{\omega \to \infty} k(\omega) = \omega^2 \cdot M \cdot |T| \cdot \cos\varphi$$

That is the same expression suggested in the test code ISO CD 10846.

3. APPLICATION

The TFM theory has been applied to the determination of the dynamic stiffness of a commercially well-known rubber engine mount: an Angst+Pfister art. 786025 type m.

As the measurement has to take into account the static preload, two different pre-loading techniques have been applied: for TFM measurement in the Z-axis (vertical), a loading mass corresponding to the engine weight was placed over the insulator; for the TFM measurement in the X-axis (horizontal), two insulators were placed face-to-face(as shown in fig. 2), with restraining screws, imposing a static deflection corresponding to the same load. In this second case, once obtained the proper deflection by careful adjusting the two screws, the flanges of the cone insulators were blocked in a jaw, placed under the vibrating plate, while a loading mass of about 10 kg was attached to a central cylinder passing through the holes of the cones.

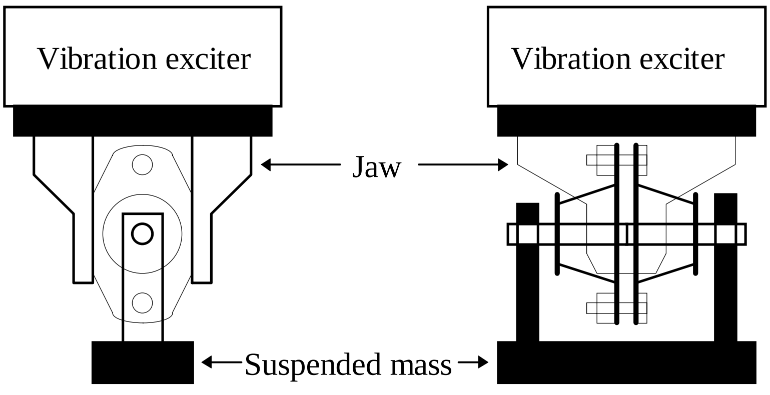

Fig. 2 - Transverse measurement with face-to-face preloading

Obviously, such a mounting gives the proper axial preload, but it gives also an unwanted transversal load, that can be minimized only reducing the suspended mass. However, the effect of this mass is usually low, because these insulators are very stiff in the transverse (X-axis) direction.

The measuring equipment is constituted of the following components:

- A two channel FFT analyzer with noise generator (OnoSokki CF920);
- A power amplifier (QsC 1500, 700 W);
- An electrodynamic shaker (Bruel & Kjaer type 4809);
- Two accelerometers (B&K type 4371);
- Two charge amplifier (B&K type 2635);

- Software to transfer the data to a PC (Onoread v. 17.2);
- A spreadsheet to performs calculations and graphic output (Quattro Pro).

In figs. 3 and 4 the (complex) transfer function data are reported for the two excitation directions. Note the peak at the mass-spring resonance frequency, and the typical phase behaviour of a single degree of freedom system. At higher frequencies, some wave effects appear. These effects give the upper frequency limit for the original Pritz theory, because he wanted to measure the rubber properties. In our case, however, these wave effect are part of the particular geometry of the specimen: they must be considered part of the specimen behaviour, and so they do not need to be disregarded.

In fig. 5 the real part of the dynamic transfer stiffness is reported: the TFM begin to work properly at about half the resonance frequency, and then extends to the higher bound of the frequency range.

Eventually, in fig. 6 the value of the loss factor η is reported: it was expected that the value of η does not vary a lot with frequency, because these rubber springs have a typical histeretical damping mechanism. Instead, the experimental results show large peak and valleys, that are probably artifacts due to bad phase matching of the transducers.

4. CONCLUSIONS

The TFM-based measure of dynamic transfer stiffness has some advantages over other proposed methods for obtaining the same results:

- Large dynamic loads need not to be generated;
- Static preload can be applied both by a proper vibrating mass and by static constraints;
- The data acquisition requires low cost hardware (compared to expensive test rigs required for the so called "direct method");
- The formulas are simple, and consistent both with the low frequency analysis, both with the high-frequency limits; the resonance region need not to be excluded from the measurement range;
- In certain cases, if a forced base motion is applicable, and the suspended mass is known, measurements can be made in situ, or however using the actual body that must be elastically supported in the reality.
- Complex measurements are made: this give useful informations on the damping of the elastic mounts (e.g. loss factor).

Also some problems must be examinated:

- It is difficult to obtain pure single axis excitation: the vibrating plate, the mounting devices, the suspended mass have to exhibit perfect simmetry to avoid spurious transverse motion or rotations; it is generally better to perform the experiment "upside down", that is with the suspended mass beneath the vibrating plate.
- A little vibration exciter cannot withstand large static loads: it is necessary to build a vibrating plate suspended by springs, and excited by the shaker, in such a way that the latter is not subjected to any static preload. Also this plate must be perfectly simmetrical, to avoid transverse motion or rotations.
- Working with random noise requires proper averaging over the Fourier spectra; the problem is alleviated using MLS pseudo random noise.

5. BIBLIOGRAPHY

[1] ISO/CD 10846 "Acoustics - Laboratory measurement of the vibro-acoustic transfer properties of resilient elements" - Part 1: Principles and guides and Part 3: Dynamic stiffness of elastic supports for translatory motion - Indirect Method.

[2] T. Pritz - "Transfer Function Method for investigating the complex modulus of acoustic materials: spring-like specimen" - Journ. of Sound and Vibration (1980) 72(3), pagg. 317-341.

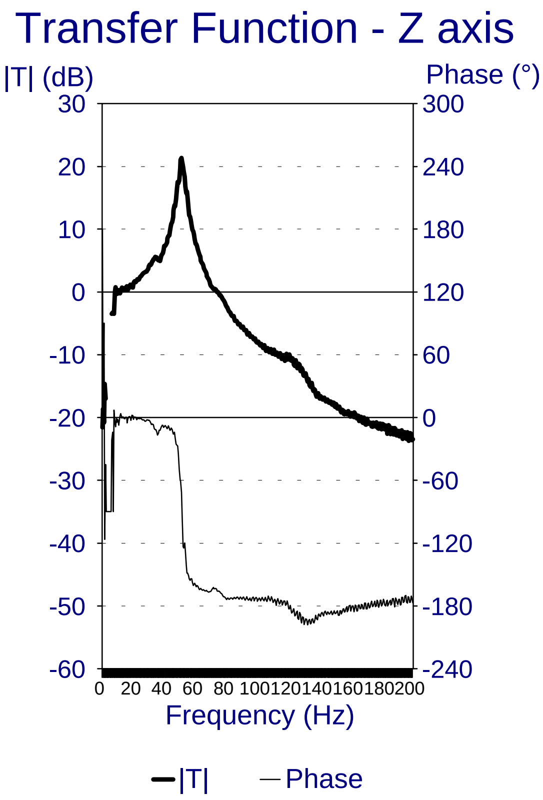

Fig. 3 - Transfer Function in Z axis

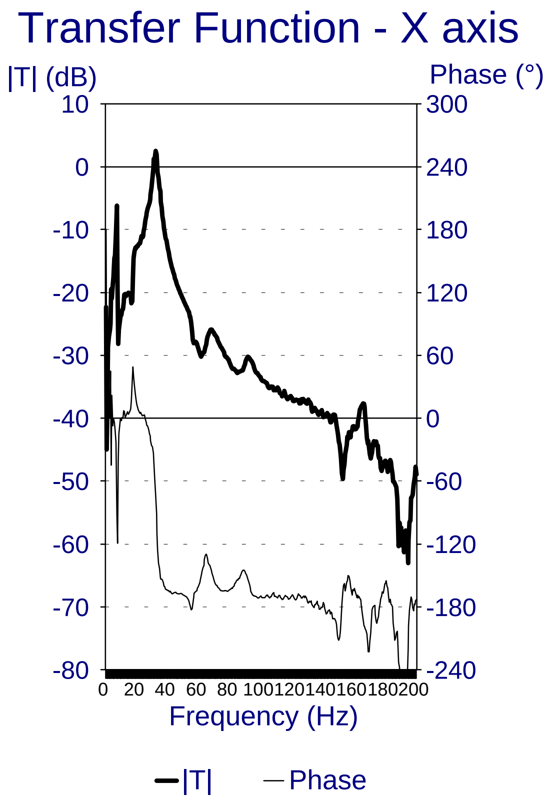

Fig. 4 - Transfer Function in X axis

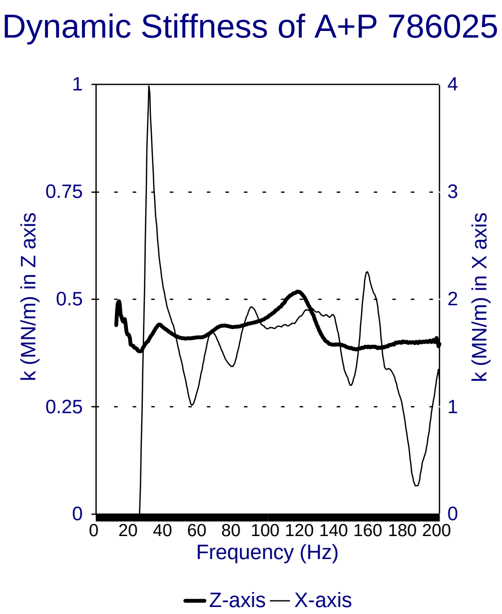

Fig. 5 - Dynamic Transfer Stiffness

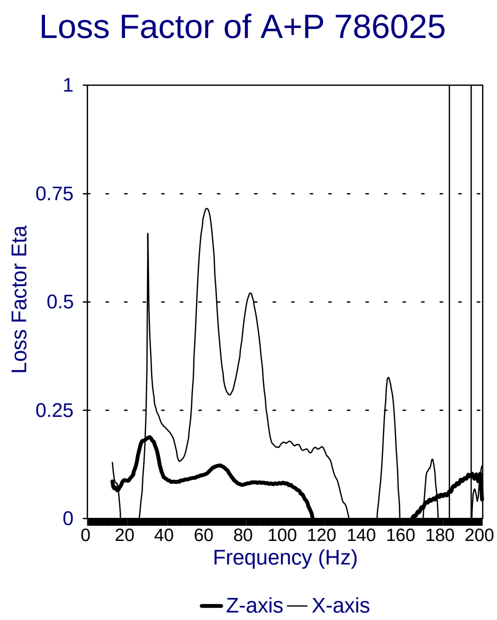

Fig. 6 - Loss Factor