

# Active control of noise by Wave Field Synthesis

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## Abstract

In this paper we present the results of a research about active control of noise. We have built a prototype to lower the direct field of a known source using a linear array of 40 loudspeakers controlled by delay lines and a digital signal processor. The inspiring theoretical tool is the Wave Field Synthesis (WFS), an holographic approach to sound control that was proposed by Berkhaut et al. in 1988 at the TU of Delft. We have obtained an average reduction of 3 db on a area of 16 squared meters for a frequency of 1000 Hz.

## 1 Introduction

Sound reduction by means of destructive interference is a well assessed methodology but, due to the physical nature of the sound field, it is very difficult to achieve sensible results over wide areas. On the other side the holographic approach of the Wave Field Synthesis to the acoustic field control is interesting because it is a tool to control the direct field of a given source on a volume. The basic idea of our research was to verify if the basic concepts of the WFS can be employed in order to build up a phase cancellation that, despite to what happens with other approaches, can be obtained over a wide area. Physically speaking we wanted to reproduce a wave front that has the same spatial properties of the one of a known source, but opposite in phase so to obtain destructive interference between the two.

In term of impulse response of the test source, this first approach is devoted to reduce the energy contained in the first 10-15 milliseconds. Such formulation of the problem should be in principle able to overcome the major difficulties that are usually encountered in traditional noise cancellation approaches, when a phase cancellation is obtained by means of a reduced number of loudspeakers placed in certain points distant from the source and usually near the receiving point where the sound reduction is required. In such a case it is well known that phase cancellation can be only a local solution, and to a certain noise reduction in a certain area corresponds a noise increase somewhere else.

We used a system devoted to control the sound on a plane (2 dimensional approximation). The array we built following the approach of the WFS has the nice property that, in the plane, irradiate very poorly outside of the angle where we expected the best reproduction.

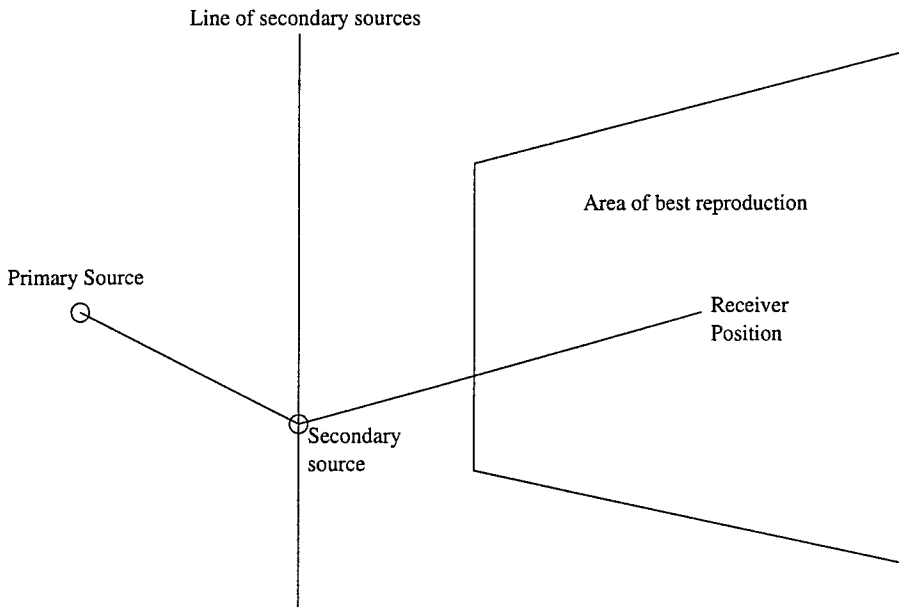


Figure 1: Two dimensional view of the geometrical setup

We made some preliminary estimations of the area where it is possible to lower the direct field of a known omnidirectional monopolar source (in the experiments we used a 8" loudspeaker). These consideration plus the way we decided to implement the anti-spatial aliasing filter suggested us the geometrical characteristics of the experimental set up.

The experiment has been performed inside an industrial "capannone" (manufacturing floor) of approximate dimensions: 50 X 20 X 8 (LxWxH).

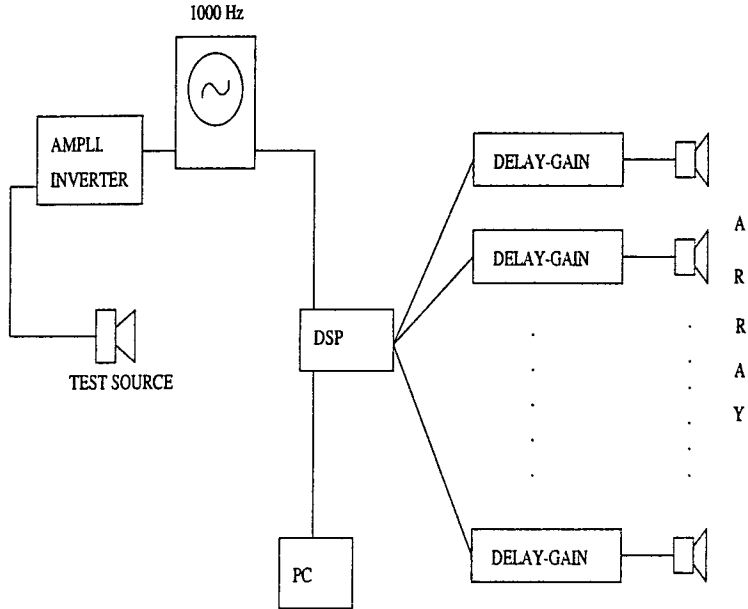


Figure 2: Block diagram of the setup

Inside that environment there were several reflecting surfaces and the reverberation time was about 2.5 sec at 1 kHz; in this way we were able to conduct our test in a situation that can be considered as good replica of a real industrial environment.

The linear array was made up of 40 4" loudspeaker units. The distance between adjacent centers of emission was 12 cm so to reach a length of about 5 meters. Array, test source, and receivers, was positioned at 2.5 meters of quota and about 5 meters from the ceiling. The Nyquist frequency regarding spatial aliasing of wave fronts that propagate in all the directions is about 1300 Hz. The distance between the source and the array was 2 meters and the area of the optimal reduction starts about 2 meters in front the array, it is large 4 meters and long at least 4. The setup has been prepared to work optimally in the octave of 1000 Hertz. The inversion of the phase in the signal of the test source has been obtained by an inverter in the the amplifier.

Each loudspeaker has tested, measuring its impulse and polar response by means of an MLSSA analyser and an array of 255 microphone's positions (see for the relative technique [3]); in this way we were able to select the loudspeakers that showed to have the most similar behaviour in the polar responses and in the temporal shape of the axial impulse response.

For figure number are depicted the axial impulse response of three loudspeakers. It can be noted that the shape in the first millisecond (where the biggest part of the energy is) is very similar.

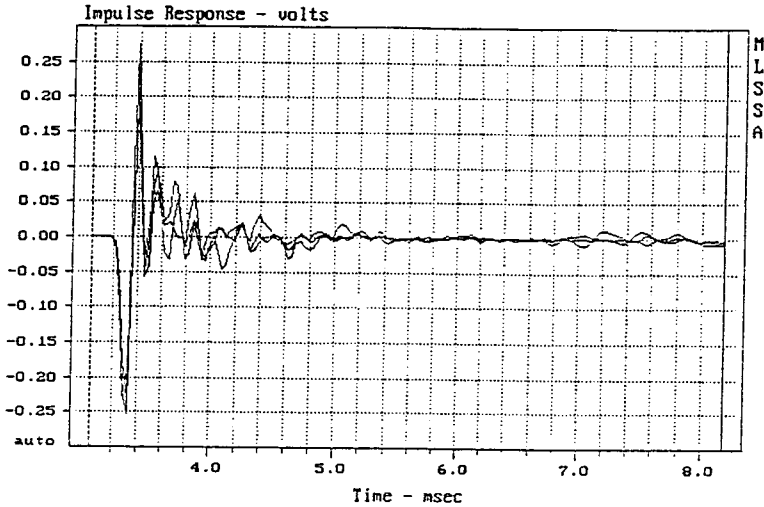


Figure 3: Axial impulse responses of three loudspeakers measured at the distance of one meter. A good homogeneity can be noted in the first millisecond.

Loudspeakers showing the less uniform characteristics were placed at the outer border of the array where the tapering <sup>1</sup>

that we applied reduced their importance in the reconstructed sound field (we'll discuss this topic in the description of the experimental set-up). Also the WFS itself, as we will see in the following, require that the loudspeakers far from the primary source (at the end of the array in our case) became less important (by means of a geometrical gain).

The use of a linear array introduce an error in the amplitude of the wave front due to the fact that the real field decay in amplitude like the inverse of the distance from the source, but the field reproduced by the array goes like one on the square root of the distance. So we decided to set the amplitude gain to obtain a correct amplitude at a certain distance from the array (in the middle of the area that we considered of "best reproduction"). Going very close to the array it is obvious that the performance of the system degrades, because a receiver is able to recognize that the sound comes from the loudspeakers of the array and not from the primary source. This approach, and the possible upgrades devoted to manage the early reflections, is two dimensional, in the sense that all the contributions

<sup>1</sup>We turned off the last 5 loudspeakers in the border following the formula  $(1 + \cos(i\pi/6))/2$ ,  $i$  is the outer loudspeaker index. ( $i=1, \dots, 5$ ).

coming from the ceiling and the floor are neglected. Often, in places like a theatre where the listeners absorb part of the energy contained in the sound that travels in the vertical direction, it is not a too bad approximation. On the other side the fact that an array of monopolar sources irradiates uniformly in all the direction equivalent under a rotation along the array, can improve the amount of these contribution. To solve this problem is under study the use of oval loudspeakers that have an higher directivity in the vertical axis than in the horizontal one.

## 2 Theory

What we are going to analyze here is a two dimensional approximation of the Kirchhoff-Helmholtz integral. The reason for this approximation is that the goal of our research is to understand how the direct field can be controlled, around a fixed quota where the heads of the listeners are supposed to stand (for example in a factory or in a theatre). Let us consider the the K-H integral (see for example [1]) in the case where the primary source is a monopole. In the formalism that we choosed  $\frac{s(t-r/c)}{r}$  is the primary source and  $S(\omega)$  the time Fourier transform of his amplitude<sup>2</sup>. The secondary sources are displaced on a straight line in front of the primary source. Under the far field conditions (see for example [4] at pg.20 and the following):

$$kr, k\Delta r \gg 1; \quad (1)$$

it is possible to obtain the so called  $2\frac{1}{2}D$  *Rayleigh Integral* for the time Fourier transform of the acoustic field, on a straight line of receivers positions in front of the line of the secondary sources:

$$P(R, \omega) = \int_{-\infty}^{+\infty} \sqrt{ik/2\pi} S(\omega) g(R, r_L) \cos(\phi_{inc}) \frac{\exp(-ikr)}{\sqrt{r}} \frac{\exp(-ikr')}{r'} dx_L; \quad (2)$$

where  $g$  is given by:

$$g(R, r_L) = \left(1 + \frac{z_L - z_S}{z_R - z_L}\right)^{-1/2}. \quad (3)$$

The geometrical quantities ( $\phi_{inc}$  included) are indicated in the figure [4]. The integrand of (2) can be written as the product of an amplitude (that we will call  $Q(x, \omega)$ ) and the monopolar propagator between the position of the secondary source and the receiver.  $Q$  is the product of different factors and it is worth to explain what they will become in the prototype:

$\sqrt{ik}$  : it will become the digital filter. The effect on the filter due to the discretization of the secondary source distribution is a multiplication for a band pass filter (different from every source) in the case we want to

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<sup>2</sup>we follow the usual convention:  $S(\omega) = \int_{-\infty}^{+\infty} \exp(-i\omega t) s(t) dt$ .

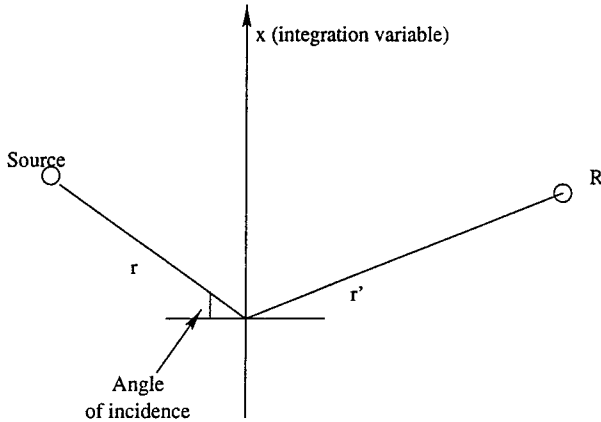


Figure 4: Geometry for the  $2^{1/2}$  Rayleigh integral

drive frequencies higher than the spatial Nyquist frequency ( $\nu = c/2\Delta x$  where  $c$  is the speed of sound and  $\Delta x$  is the spacing between the loudspeakers).

$S(\omega)$  : spectrum of the monopolar test source (in our prototype is a delta around 1000 Hz.).

$\cos(\phi_{inc})/\sqrt{r}$ : This will become a geometrical gain implemented by an analogical (in this first prototype) board.

$\exp(-ikr)$  : geometrical delay controlled by bucket brigade delay lines.

$g(R, r_L)$  : For this term we write a more detailed digression.

Due to  $g$  the equation (2) has an unpleasant aspect: the amplitude (note: **not the phase!**) of the secondary sources depends on the position of the receiver  $R$ . This is a problem because we would like that the amplitude depends only on the position of the source and the geometrical characteristics of the system (position and shape of the array and area where we want to reproduce as correctly as possible the field). A solution that works can be found, because at this stage, our goal is not to cancel exactly the direct field, but estimate the area where it is possible to lower enough the direct sound level. From here to the following we will concentrate our attention in a reduction of 8 dB.<sup>3</sup> To this goal let us note that if we sum to a given primary field ( $P_1$ ) a secondary one ( $P_2$ ) that is in counter-phase and with

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<sup>3</sup>Our first prothotype works only on the direct field and we do not aim, at this stage to obtain an higher reduction.

a different amplitude

$$P_1 = A\cos(\phi), \quad P_2 = B\cos(\phi + \pi) \quad (4)$$

$$P_1 + P_2 = A(1 - A/B)\cos(\phi) \quad (5)$$

we can observe that the control of the amplitude, to obtain a good reduction, is not very critical. In fact from the expression of the lowering respect the primary one:

$$\Delta L = L_{P_1} - L_{P_1+P_2} = 10 * \text{Log}((1 - A/B)^2) \quad (6)$$

it is easy to see that to lower of 8 dB a given field (using another field exactly in counter phase) is allowed an error in amplitude up to 40 %.

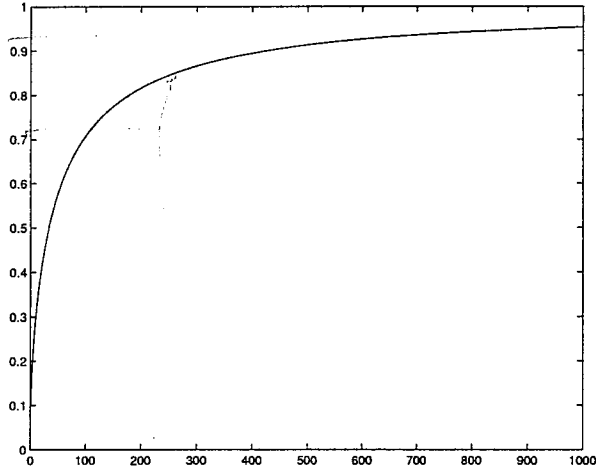


Figure 5:  $g$  plotted in function of  $\chi = \frac{z_L - z_S}{z_R - z_L}$

From figure (5), where we plotted  $g$  in the form (2) in function of  $\chi = \frac{z_L - z_S}{z_R - z_L}$ , is possible to note that if we set  $g$  for a line of receivers that stands 4 meters away from the array ( $\chi = 2$ ) the condition of not exceeding the error of 40 per cent in amplitude is respected about from 1,5 meter of distance from the array.

### Spatial Aliasing

Spatial aliasing (SA) is one of the problems that has to be faced in practical realizations of a system that controls the spatial properties of the acoustic field. To explain intuitively the effect of SA, let us observe that the distance between adjacent loudspeakers ( $\Delta x$ ) can be used to define

a scale of frequencies ( $\nu_{Nyquist} = c/2\Delta x$ ). SA is an error inducted by the array that emerges when we try to reproduce wave-packets containing high frequencies (compared with  $\nu_{Nyquist}$ ), that move transversal to the line of the loudspeakers. The Nyquist condition, that can be derived substituting a line distribution of sources with an infinite array of equal-spaced omnidirectional punctual sources

$$|k_x| \leq \pi/\Delta x; \quad (7)$$

means that if the spatial Fourier transform of the amplitude of the monopolar distribution of sources  $Q(k_x)$ <sup>4</sup> has no component with  $|k_x| \geq \pi/\Delta x$  the field is correctly reproduced by the array.

### How many sources in the array?

Another issue that we would like to mention regards the number of sources. In fact if only few sources are used there are limitations in the reproduction of the spatial properties of the field (see for example [8]).

These limitations have a different nature respect the case of the spatial aliasing. SA is an undesired effect caused by the implementation of the continuous line of secondary sources with a “discrete” number of loudspeakers. As explained above the limitations are on wave-packets that propagate transversally to the array. The limitation that emerges when we use an array composed by few loudspeakers, on the other side, regards more the length of the array and concerns the reproduction of wave-packets that move perpendicular to the array. For example we can't reproduce the spatial properties of a plane wave with an omnidirectional point source (one secondary source case).

In general if you want to reproduce a plane wave and you position a receiver at a distance equal or larger than the diameter of the array the wave fronts start to become curved (and not flat). Physically means that if we want to reproduce wave fields with no components for  $k_x \neq 0$  we obtain a field that has these components. Let us consider, for example, an array of  $2M + 1$  loudspeakers equally spaced ( $\Delta x$ ). The spatial transform of the amplitude of the secondary sources ( $Q(k_x, t)$ ), has to be convolved with the functional<sup>5</sup>

$$\Delta(k_x) = 1 + \sum_{n=0}^M \cos(k_x n \Delta x); \quad (8)$$

This term is given by the Fourier transform of a sum of a finite number of delta functions located at the positions of the loudspeakers (see for example [5]). We chose a pair number of secondary sources because in the limit

<sup>4</sup> $Q(k_x)$  is the spatial Fourier transform of  $Q(x, z, \omega)$  respect  $x$ .

<sup>5</sup> $\Delta(k_x)$  in general is a functional (and not function) in the sense that performing the limit  $M \mapsto \infty$ , it has a sense only inside the integral.



$M \mapsto 0$  the array goes in the "one source" case. to derive (6) we took the origin of the  $x$  axis on the central secondary source. When  $M \mapsto \infty$  the functional  $\Delta(k_x)$  converges towards a sum of infinite Dirac's deltas translated of a multiple of the quantity  $2\pi/\Delta x$  that is the double of the spatial Nyquist frequency. In this limit the spatial spectrum of  $Q(k_x, t)$  became simply periodized, by the convolution with  $\Delta(k_x)$  and the standard ideas on the anti-aliasing filtering lead to a correct implementation of the field.

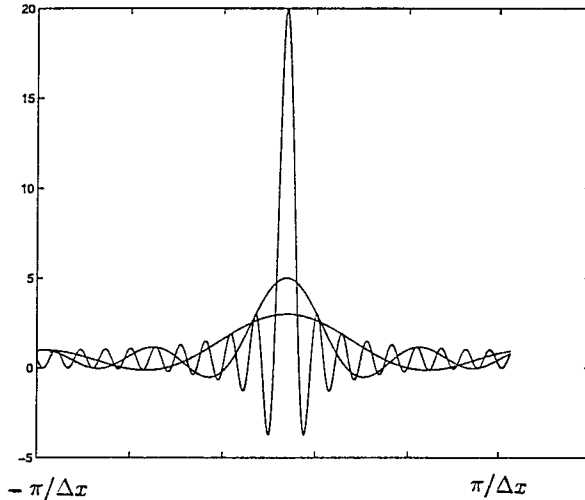


Figure 6:  $\Delta(k_x)$  plotted in the Nyquist region for 5,9,39 loudspeakers

In real cases  $M$  is finite and so  $\Delta(k_x)$  is not a sum of Dirac's delta. In figure(1) we have plotted  $\Delta(k_x)$  for tre different numbers of loudspeakers. It is clear that if we use a low number of loudspeakers, the convolution between  $Q(k_x, t)$ , and  $\Delta(k_x)$  changes the shape of  $Q(k_x, t)$  inside the Nyquist zone, (even in the case  $Q(k_x, t) = 0$  for  $|k_x| > \pi/\Delta x$ ). We can imagine the curves in the figure as the result of a convolution of a sinc (the Fourier transform of a squared window that has the lenght of the array), with the sum of Dirac's deltas (of the infinite array). In our case (40 loudspeakers) can be proved (we have work in progress on it) that this effect is not the main source of error <sup>6</sup>.

### Matlab simulations

Some matlab simulation have been performed to get a qualitative insight

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<sup>6</sup>This arguments are extensively covered in the litterature of radar and sonar processing.

on the performance of the array The sources of the array have been considered monopolar with a delta impulse response. We have considered a primary source emitting a sinc containing the frequencies of the octave of 1000 hz. We have calculated the field on a grid of points in front the array. the grid is large 8 meters and long 4. the pass of the grid is 0.5 cm. In fig 7 is depicted a configuration at a fixed time of the sum of the two field. In the area that we considered of best reproduction the primary it is possible to note the partial cancellation of the primary field. A good property is that outside is not amplified (due to the directivity of the array). If we take one slice of the configuration in the central zone ( $x=800$ ) and plot the two fields we can obtain the figure 8 that suggest an upper limit in the reduction of 20 dB for this range of frequencies.

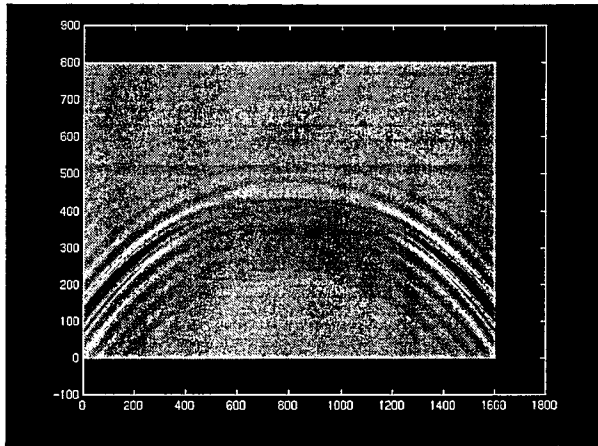


Figure 7: Matlab simulation: this is a map, at a fixed time of the sum of the primary and secondary field. The primary one is a sinc containing frequencies in the octave of 1000 Hz. The area (located two meters from the array) is 8 meters wide and 4 meters long. The grid of the figure has a step of 0.5 cm. In the central area the two fields smooth each other, and in the sides the level the primary field is not highered.

### 3 Experimental Setup

Some remark can be made about the experimental set up. The goal is to show how some practical problem have been faced and how the performance of the system can be improved.

- Delay lines (Bucket brigade, model MN 3004) controlled by a two phase

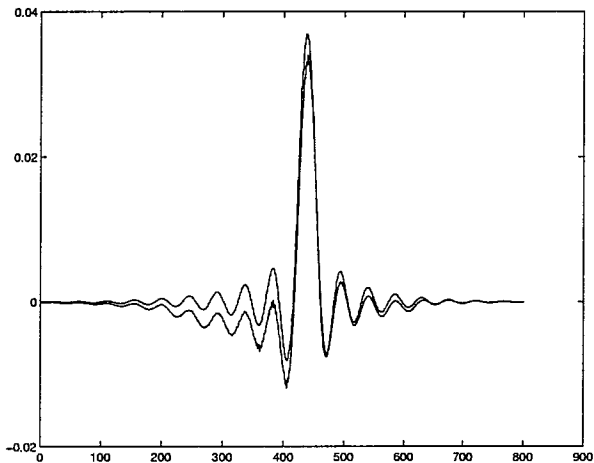


Figure 8:  $\Delta(k_x)$  Primary and secondary field plotted on a line perpendicular to the array from a distance of 2 meters until 6 meters away.

clock generator (MN3101) have been used in order to set the geometrical delay required for the signals to be driven to the loudspeakers of the array. The clock generators have shown an **intrinsic instability** in maintaining the frequency so the delay of the signal have probably become randomized and the reproduction has probably suffer of it. Is important that we have obtain a positive result even in the presence of this weakness. Future prototypes will be developed including the geometrical contribution to delay and gains inside the work of the digital signal processor.

-Loudspeakers interact between eachother. For example we installed three loudspeaker in the wood structure of the array and mesured the axial impulse responses of each loudspeaker mantaining the other two turned off (see fig (9)). The energy contained in each IR was bigger than the energy of a loudspeaker alone. The speaker with the other two turned off positioned in the two lateral sides, had an impuse response containig an energy bigger than the other two. This is an effect of the fact that the copuling depends on the distance. From the figure (9) can be noted that for wavelenght short respect the spacing:

$$\lambda \ll Dx \tag{9}$$

the copuling vanish. A tecnique to analize in a quantitative way the spatial properties of the effective sources is discussed in [3]. What we expected was that installing the more omogeneous sources in the centre of the array they interact giving omogeneous “effective” sources.

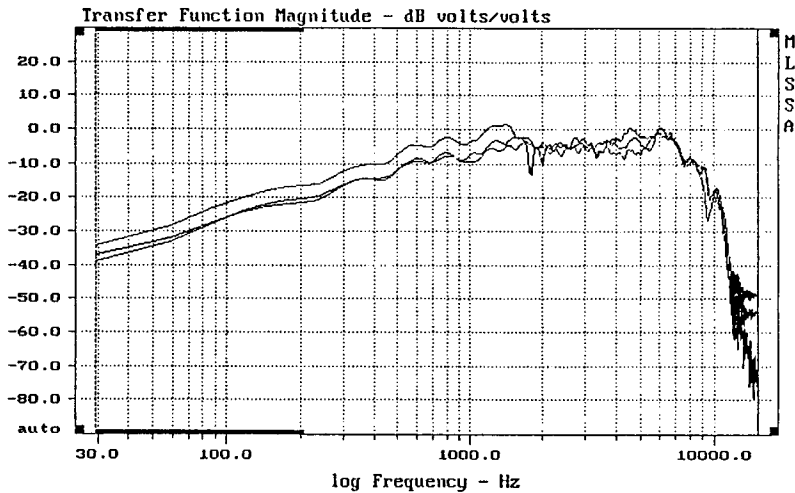


Figure 9: Frequency content of the impulse responses of three adjacent loudspeakers in the array. Every measurement of the impulse response is performed taking one of them turned on and the others turned off. The most energetic one is the central loudspeaker.

was not like an array of monopolar sources, but probably more close to a dipolar one. Closing the back of the array should lead to a more “monopolar” emission (fig 10, B). The reproduction has probably suffer also of this fact, especially for wide angle respect the line that connect the source with the center of the array.

## 4 Experimental results

The experiment have been performed giving to the test source and the the electronic system that pilot the array the same signal. The gain of the test source has been adjusted to to give the the same pressure level in the in the center of the receivers area. Unfortunately the first test source we used was able to give a 90 dB level when the signal of the array was 91.8 dB. This is another point were the performance of the can be improved. The result of the mesurments of SPL performed by a BruelKjaer 2236 Integrating Sound Level Meter, are depicted in the figure num. The important aspect is that the reduction is on a wide area as we desired.

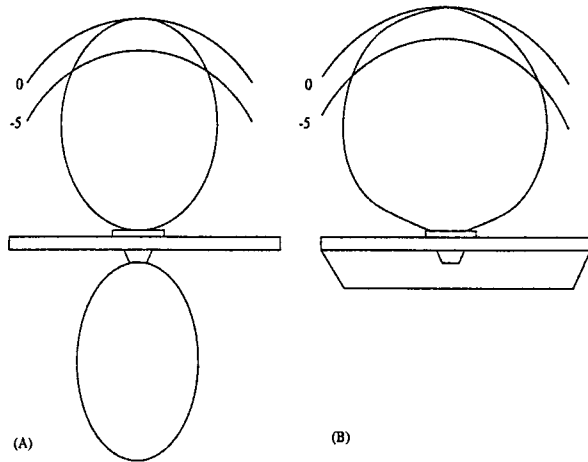


Figure 10: Qualitative description of the expected modification in the polar patterns of the speakers of the array, closing the back of the array.

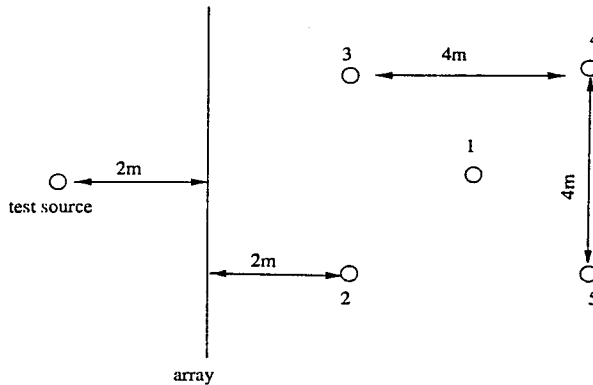


Figure 11: position of the experimental measurements

## 5 Conclusions

We have analyzed some necessary issues to understand how to cancel the direct field of a known source. We have also analyzed some limitations (most of them easy solvable) of our first prototype. The fact that the experimental results were positive (even in the presence of these limitations) is an interesting point. An aspect that we should remark is that the reduction has been obtained on a wide area. The main goal of this research is to design a low-cost device to be employed in industrial environments in order

POSITION	PRIMARY LEV.	ARRAY LEV.	SUM	LOWERING (dB.)
1	90.1	91.8	85.9	-4.23
2	91.6	93.2	90.1	-1.14
3	91.4	93.1	90.4	-1
4	88.4	90.1	86	-2.46
5	88.3	90.2	85.8	-2.5

Figure 12: Mesured data

to obtain sensible noise reduction over large portion of space, but other practical implementations are currently under investigation too. Research to control early reflections and wide band signal are on progress.

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