

## 12. VIBRATION ISOLATION

### 12.1 Introduction

High vibration levels can cause machinery failure, as well as objectionable noise levels. A common source of objectionable noise in buildings is the vibration of machines that are mounted on floors or walls. Obviously, the best place to mount a vibrating machine is on the ground floor. Unfortunately (but fortunately for noise control consultants), this is not always possible. A typical problem is a rotating machine (such as a pump, AC compressor, blower, engine, etc) mounted on a roof, or on a floor above the ground floor. The problem is usually most apparent in the immediate vicinity of the vibration source. However, mechanical vibrations can transmit for long distances, and by very circuitous routes through the structure of a building, sometimes resurfacing hundreds of feet from the source. A related problem is the isolation of vibration-sensitive machines from the normally occurring disturbances in a building (car or bus traffic, slamming doors, foot traffic, elevators...). Examples of sensitive machines include surgical microscopes, electronic equipment, lasers, MRI units, scanning electron microscopes, and computer disk drives.

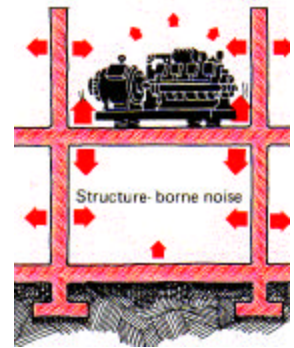


Figure 1 shows a common example of a vibration source, a large reciprocating air conditioning compressor weighing 20,000 pounds, mounted on a roof. Annoying noise levels at multiples of the compressor rotational frequency, predominantly 60 and 120 Hz, were measured in the rooms directly below the compressor. For whatever reason (don't get me started...), the architect chose to mount this unit at the middle of the roof span, at the midpoint between supporting columns. Also, this type of compressor (reciprocating) is notorious for high vibration levels. Centrifugal or scroll type compressors are much quieter, but more expensive (you get what you pay for!).



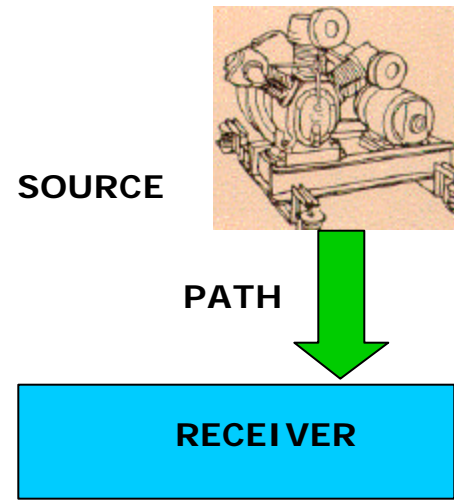
Figure 1. A reciprocating air conditioning compressor and chiller mounted on a flexible roof, Note the straight conduit on the left which bypasses the isolators and directly transmits vibration into the roof

A vibration problem can also be nicely described by the same *source – path – receiver* model we previously used to characterize the noise control problem.

**Source:** a mechanical or fluid disturbance, generated internally by the machine, such as unbalance, torque pulsations, gear tooth meshing, fan blade passing, etc. These typical occur at frequencies which are integer multiples of the rotating frequency of the machine.

**Path:** the structural or airborne path by which the disturbance is transmitted to the receiver

**Receiver:** the responding system, generally having many natural frequencies which can potentially be excited by vibration frequencies generated by the source. (Murphy says the natural frequency of the system will **always** coincide with an excitation frequency.)



Any or all of these areas can be attacked to solve the problem. The best choice for a given application will be dictated by the laws of physics, your ingenuity, and \$.

**References:** Sources of additional information on vibration isolation include:

- Manufacturer's technical data – check the web or catalog data sheets, some suppliers include:
  - Mason Industries
  - Barry Controls
  - Kinetics
- *ASHRAE (American Society of Heating, Refrigeration and Air Conditioning Engineering) Applications Handbook*, 1999. See Appendix
- *Handbook of Acoustical Measurements and Noise Control*, Cyril M. Harris, McGraw Hill, 1991.
- *Noise and Vibration Control Engineering*, L. Beranek editor, John Wiley and Son, 1992, pp 429-450

## 12.2 Possible Solutions

The best solution to a vibration problem is to avoid it in the first place. Intelligent design is far more cost effective than building a bad design and having to repair it later. The intelligent solution to any vibration problem involves the following steps:

- 1) Characterize the system parameters (mass, stiffness, damping) by experimental methods, manufacturers data, or a combination of both.
- 2) Model the system dynamics using a simple lumped parameter model
  - a) identify natural frequencies, look for coincidence with excitation frequencies
  - b) if excitation forces and frequencies are known, system response can be calculated
- 3) Use the model to assess the effect of changes in system parameters

### Vibration Solutions - Source

- 1) Relocate machine – place machine on as rigid a foundation as possible (on grade is best) and as far as possible from potential receivers
- 2) Replace machine with a higher quality or different type of machine that is quieter (and probably more expensive)
- 3) Change the operating speed of the unit to avoid coinciding with structural resonances
- 4) Balance rotating elements,
- 5) Add a tuned vibration absorber
- 6) Use active vibration control

### Vibration Solutions - Path

Minimizing the vibration transmission generally involves using isolator springs and/or inertia blocks. The basic principle is to make the natural frequency of the machine on its foundation as far below the excitation frequency as possible. The mathematics for this case, and isolator selection procedures are discussed in the next sections.

### Vibration Solutions – Receiver

- 1) Change the natural frequencies of the system to avoid coinciding with excitation frequencies. This can be accomplished by adding stiffeners (which raises the natural frequency) or by adding mass (which lowers the natural frequency)
- 2) Add structural damping

## 12.3 Vibration Isolators

Consider a vibrating machine, bolted to a rigid floor (Figure 2a). The force transmitted to the floor is equal to the force generated in the machine. The transmitted force can be decreased by adding a suspension and damping elements (often called vibration isolators) Figure 2b , or by adding what is called an inertia block, a large mass (usually a block of cast concrete), directly attached to the machine (Figure 2c). Another option is to add an additional level of mass (sometimes called a seismic mass, again a block of cast concrete) and suspension (Figure 2d).

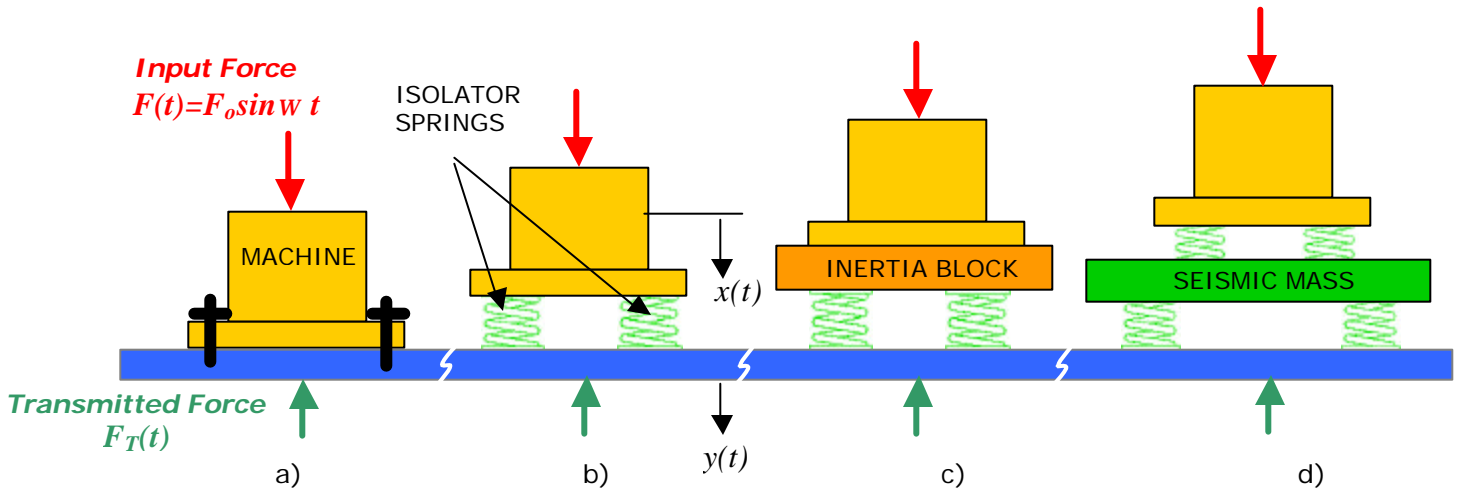


Figure 2. Vibration isolation systems: a) Machine bolted to a rigid foundation b) Supported on isolation springs, rigid foundation c) machine attached to an inertial block d) Supported on isolation springs, seismic mass and second level of isolator springs

If we consider only the vertical motion, the case shown in Figure 2b can be described mathematically by a single degree of freedom, lumped element system.

$$m \ddot{x} + c \dot{x} + kx = F(t) \quad \text{Equation 1}$$

where:

$m$  = mass of system     $k$  = stiffness     $c$  = viscous damping

$x(t)$  = vertical displacement     $F(t)$  = excitation force

If we neglect damping, the vertical motion of the system,  $x(t)$  can be shown to be:

$$x(t) = \frac{F_0/k}{1-r^2} \sin wt \quad \text{where:} \quad r = \frac{w}{w_n} \quad w_n = \sqrt{\frac{k}{m}} \quad \text{Equation 2}$$

The system has a natural, or resonant frequency, at which it will exhibit a large amplitude of motion, for a small input force. In units of Hz (cycles per second), this frequency,  $f_n$  is:

$$f_n = \frac{w_n}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad \text{Equation 3}$$

In units of RPM (revolutions per minute), the critical frequency is

$$RPM_{critical} = 60f_n = \frac{60}{2\pi} \sqrt{\frac{k}{m}}$$

The force transmitted to the floor is:  $F_T = kx$

The ratio of transmitted force to the input force is called **transmissibility**,  $T$

$$T = \left| \frac{F_T}{F_0} \right| = \left| \frac{1}{r^2 - 1} \right| = \left| \frac{X}{Y} \right| \quad \text{Equation 4}$$

This same equation can also be used to calculate the response of a machine  $X$  to displacement of the foundation,  $Y$ .

The effectiveness of the isolator, expressed in dB is:

$$E = 10 \log_{10} \frac{1}{T} \quad \text{Equation 5}$$

The effectiveness of the isolator, expressed in percent is:

$$\% \text{ Isolation} = (1 - T) * 100 \quad \text{Equation 6}$$

The transmissibility as a function of frequency ratio is shown in Figure 3. Vibration isolation (defined as  $T < 1$ ) occurs when the excitation frequency is  $> 1.4 f_n$ . For minimum transmissibility (maximum isolation), the excitation frequency should be as high above the natural frequency as possible. The transmissibility above resonance has a slope of  $-20$  dB/decade.

The transmissibility including the effect of damping is:

$$T = \frac{\sqrt{1 + (2xr)^2}}{\sqrt{(1 - r^2)^2 + (2xr)^2}} \quad \text{Equation 7}$$

$$\text{where : } x = \frac{c}{2m\omega_n} = \text{critical damping ratio}$$

Typical values for damping ratio,  $\xi$  are .005 -.01 for steel, and .05-.10 for rubber

The inclusion of damping has the greatest effect in the vicinity of resonance, decreasing the vibration amplitude. A curious effect of damping is that it results in **increased** amplitude at frequencies  $> 1.4 f_n$ .

**Examples:**

- 1) Calculate the transmissibility at 60 and 120 Hz for a 20,000 lb chiller unit supported by eight springs with 3" static deflection. Answer:

Frequency – Hz	r	T	dB	% Isolation
60	60/1.8 = 33.33	.0009	30.5	99.910%
120	120/1.8	.00022	36.6	99.978%

- 2) A surgical microscope weighing 200 lb is hung from a ceiling by four springs with stiffness 25 lb/in. The ceiling has a vibration amplitude of .05mm at 2 Hz (a typical resonant frequency of a building). How much vibration does the microscope experience? Answer:  $r = .903$ ,  $|X/Y| = 5.45$ ,  $X = .273$  mm (we have amplification)

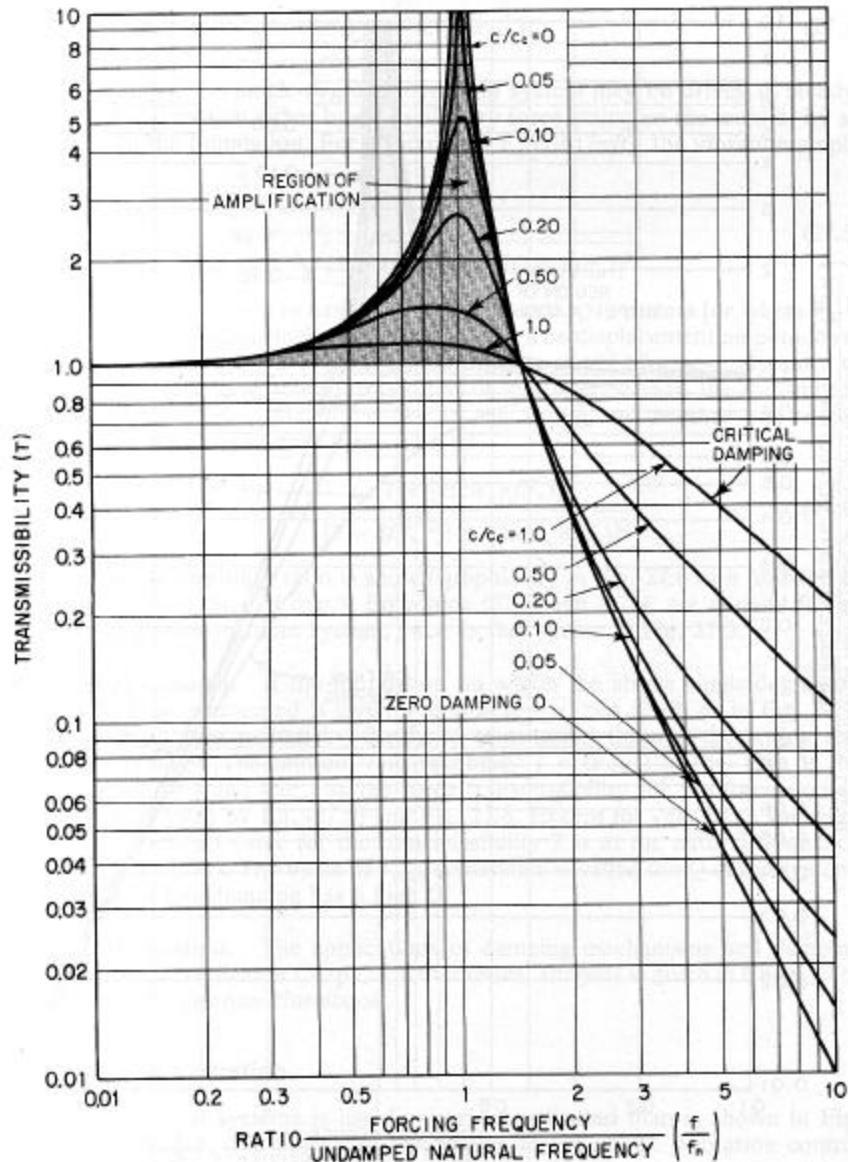


Figure 3. Force or displacement transmissibility for a viscously damped single degree of freedom system

Typical vibration isolators employ a helical spring to provide stiffness, and an elastomeric layer (such as neoprene) to provide some damping. Other types use a solid elastomeric element for both the stiffness and the damping. Some commercially available isolators are shown in Figures 4 and 5. Isolators are available for tension applications (for hanging equipment, such as pipes) or compression. When even lower stiffnesses are required than can be obtained with coil springs, pneumatic springs can be employed. The stiffness of pneumatic springs is controlled by their inflation pressure.

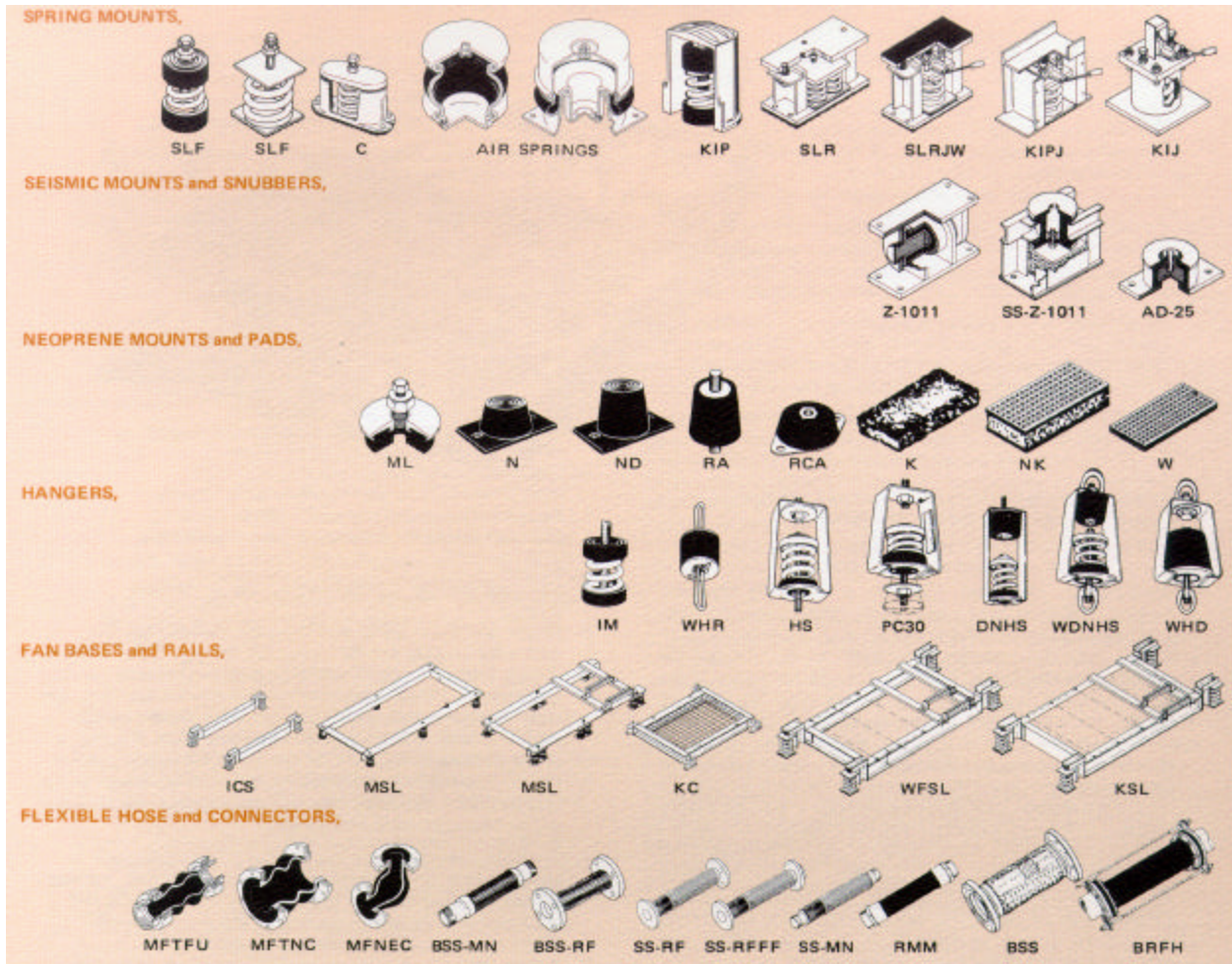


Figure 4. Off the shelf vibration isolation products from Mason Industries Inc.



Figure 5. Examples of commercially available elastomeric isolators from Barry Controls

## 12.4 Isolator Selection

Isolators are usually specified by their static deflection  $\Delta$ , or how much they deflect when the weight of the machine is placed on them. This is equivalent to specifying their stiffness and has the additional benefit of making it easy to calculate the system natural frequency. Coil spring isolators are available in up to 3" static deflection. If more flexibility is needed, air springs are used. The natural frequency of the system (assuming a single degree of freedom) can be calculated by:

$$f_n = \frac{1}{2p} \sqrt{\frac{k}{m}} = \frac{1}{2p} \sqrt{\frac{kg}{W}} = \frac{1}{2p} \sqrt{\frac{g}{\Delta}} = 3.13 \sqrt{\frac{1}{\Delta}} \text{ Hz} \quad \text{Equation 8}$$

where:  $\Delta$  = static deflection of spring (inches)  
 $g$  = gravitational constant = 386 in/sec<sup>2</sup>

Static Deflection (inches)	0.5"	1.0"	2.0"	3.0"
Natural Frequency - Hz	4.43 Hz	3.13 Hz	2.21 Hz	1.8 Hz

**Example:** A 400 lb duct is to be hung from a ceiling. 30 dB of isolation is desired at all frequencies greater than 100 Hz. Determine the desired stiffness, and static deflection of each isolator spring if four springs are to be used. (Answer: assuming all four springs are equally loaded,  $K=102$  lb/in,  $\Delta= .98$ " )

## 12.5 Important Considerations with Vibration Isolator Selection

### 1) Machine Location

- As far away from sensitive areas as possible
- And on as rigid a foundation as possible (on grade is best)

### 2) Proper sizing of isolator units

- Correct stiffness (specified by the static deflection, more flexible is generally better)
- Sufficient travel to prevent bottoming out during shock loads, or during system startup and shutdown

**3) Location of isolators** – isolators should be equally loaded, and the machine should be level.

**4) Stability** – sideways motion should be restrained with snubbers. The diameter of the spring should also be greater than its compressed height. Isolator springs should occupy a wide footprint for stability.

**5) Adjustment** – springs should have free travel, should not be fully compressed, nor hitting a mechanical stop

**6) Eliminate vibration short circuits** – any mechanical connection between machine and foundation which bypasses the isolators, such as pipes, conduits, binding springs, poorly adjusted snubbers or mechanical stops

**7) Fail safe operation** – should a spring break or become deflated, you must have mechanical supports on which the machine can rest without tipping.

Additional guidance in isolator selection can be found in the ASHRAE Applications Handbook, included as an appendix.



## 12.6 Vibration Units

In addition to the usual confusion between English and SI units, there is also no universal standard for what vibration quantity to measure – displacement, velocity, or acceleration. Different vibration standards use different units. The ASHRAE standards in Figure 6, for instance, use rms velocity (in/sec).

The most common measurement device is the accelerometer, which as its name would suggest, provides a voltage output proportional to the vibration acceleration (for example, 10mV/g). Velocity or displacement are obtained by integrating the acceleration signal once or twice respectively. This integration can be done electronically, or numerically. Integration is equivalent to dividing by the frequency,  $\omega$  (radians/sec). To illustrate the possibilities and the necessary conversions, consider an example system with an actual physical movement of  $\pm 1$ mm, vibrating at 10 Hz ( $\omega = 2\pi 10 = 62.83$  rad/sec):

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**Displacement:**  $X(t) = |X| \sin \omega t = 1.0 \sin 2\pi 10t$  Equation 9

Displacement magnitude:  $|X| = 1.0 \text{ mm } (.039 \text{ in})$

RMS (root mean square) displacement:  
 $X_{\text{rms}} = .707 |X| = .707 \text{ mm } (.028 \text{ in})$

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**Velocity:**  $V(t) = \frac{d}{dt} X(t) = \omega |X| \cos \omega t = |V| \cos \omega t$  Equation 10

Velocity magnitude:  $|V| = \omega |X| = 62.83 \text{ mm/sec } (2.45 \text{ in/sec})$

RMS velocity:  $.707 |V| = 44.42 \text{ mm/sec}^2 (1.73 \text{ in/sec})$

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**Acceleration:**  $A(t) = \frac{d}{dt} V(t) = -\omega^2 |X| \sin \omega t = |A| \sin \omega t$  Equation 11

Acceleration magnitude:  $|A| = \omega^2 |X| = 3948 \text{ mm/sec}^2 (153.9 \text{ in/sec}^2)$

and since  $9.8 \text{ m/sec}^2 = 1 \text{ g}$ ,  $0.40 \text{ g}$   
 RMS acceleration:  $.707 |A| = 2791 \text{ mm/sec}^2 (108.8 \text{ in/sec}^2)$

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On the other hand, if acceleration amplitude  $|A|$  is known, velocity and displacement can be calculated by:

$$|V| = \frac{1}{\omega} |A| \quad \text{and} \quad |X| = \frac{1}{\omega^2} |A| \quad \text{Equation 12}$$

## 12.7 Severity of Vibration

How much vibration is too much? Recommended vibration levels for various applications are shown in Figure 6 (reference: ASHRAE Applications Handbook, 1999). A more complete excerpt from the ASHRAE handbook is included as an Appendix.

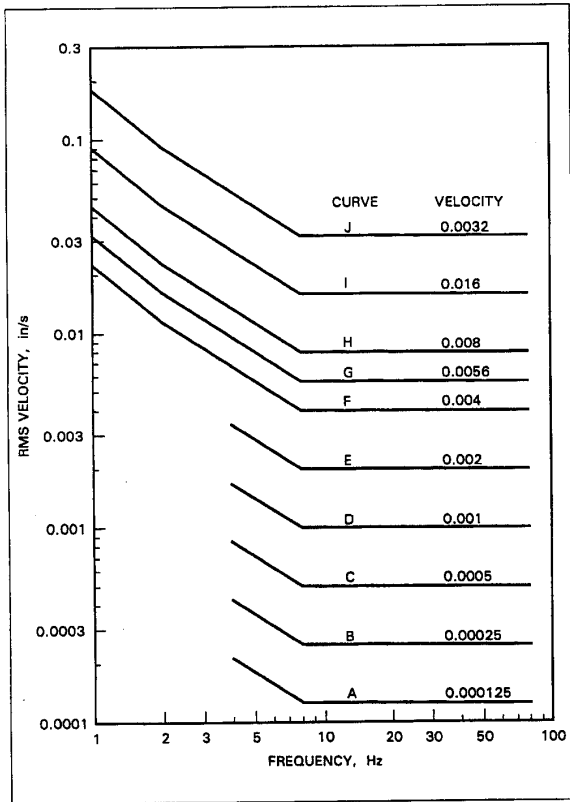


Fig. 40 Building Vibration Criteria for Vibration Measured on Building Structure

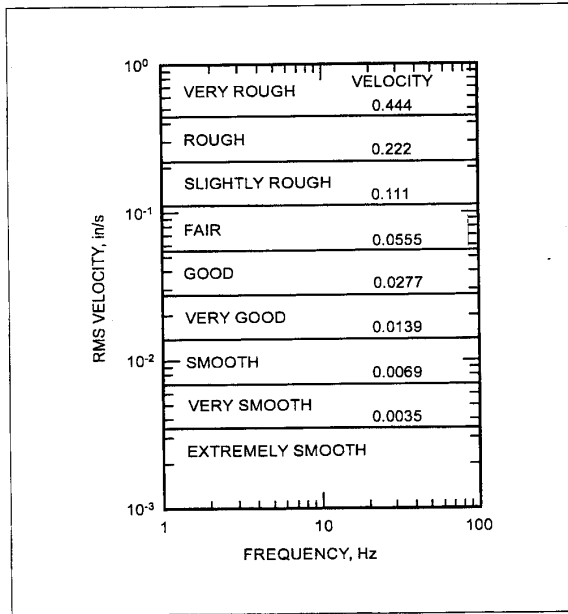


Fig. 41 Equipment Vibration Severity Rating for Vibration Measured on Equipment Structure or Bearing Caps

Table 43 Equipment Vibration Criteria

Human Occupancy	Time of Day	Curve <sup>a</sup>
Workshops	All	J
Office areas	All <sup>b</sup>	I
Residential (good environmental standards)	0700-2200 <sup>b</sup>	H-I
	2200-0700 <sup>b</sup>	G
Hospital operating rooms and critical work areas	All	F

Equipment Requirements	Curve <sup>a</sup>
Computer areas	H
Bench microscopes up to 100× magnification; laboratory robots	F
Bench microscopes up to 400× magnification; optical and other precision balances; coordinate measuring machines; metrology laboratories; optical comparators; microelectronics manufacturing equipment—Class A <sup>c</sup>	E
Microsurgery, eye surgery, neurosurgery; bench microscope at magnification greater than 400×; optical equipment on isolation tables; microelectronic manufacturing equipment—Class B <sup>c</sup>	D
Electron microscopes up to 30,000× magnification; microtomes; magnetic resonance imagers; microelectronics manufacturing equipment—Class C <sup>c</sup>	C
Electron microscopes at magnification greater than 30,000×; mass spectrometers; cell implant equipment; microelectronics manufacturing equipment—Class D <sup>c</sup>	B
Unisolated laser and optical research systems; microelectronics manufacturing equipment—Class E <sup>c</sup>	A

<sup>a</sup>See Figure 40 for corresponding curves.  
<sup>b</sup>In areas where individuals are sensitive to vibration, use curve H.  
<sup>c</sup>Classes of microelectronics manufacturing equipment:  
 Class A: Inspection, probe test, and other manufacturing support equipment.  
 Class B: Aligners, steppers, and other critical equipment for photolithography with line widths of 3 μm or more.  
 Class C: Aligners, steppers, and other critical equipment for photolithography with line widths of 1 μm.  
 Class D: Aligners, steppers, and other critical equipment for photolithography with line widths of 0.5 μm; includes electron-beam systems.  
 Class E: Aligners, steppers, and other critical equipment for photolithography with line widths of 0.25 μm; includes electron-beam systems.

Table 44 Equipment Vibration Criteria

Equipment	Allowable rms Velocity, in/s
Pumps	0.13
Centrifugal compressors	0.13
Fans (vent sets, centrifugal, axial)	0.09

Figure 6. Recommended vibration levels for various applications (ref. ASHRAE Applications Handbook, 1999, page 46.38)

## 12.8 Surging of Coil Springs

The theoretical isolation obtained by Equation 4 assumes a single degree of freedom system, i.e., that there are no other natural frequencies. At high frequencies, a coil spring can have an internal resonant frequency, at which it will effectively transmit vibration. The addition of an elastomer layer between the spring and its support will attenuate transmission at this frequency. Commercial coil spring isolator units will generally have a layer of neoprene above or below the coil to minimize this problem.

This natural frequency (sometimes called the “surge” frequency) can be calculated by:  
(reference: *Mechanical Engineering Design, Shigley and Mitchell, 1983*)

$$f_n = \frac{n}{2} \sqrt{\frac{kg}{W}} \quad \text{Hz} \quad \text{Equation 13}$$

where:  $n$  = integer mode #, 1,2,3...

$k$  = stiffness of spring

$g$  = gravitational constant (386 in/sec<sup>2</sup>)

$$W = \text{weight of the spring (active coils)} \quad W = n_c \frac{\rho d^2}{4} \rho D r$$

$n_c$  = number of active coils

(for a spring with ground, flattened ends,  $n_c$  = total number of coils – 2)

$D$  = mean diameter of coil

$d$  = diameter of wire

$r$  = weight density of material (.283 lb/in<sup>3</sup> for steel)

The stiffness of a coil spring,  $k$  can be calculated by:

$$k = \frac{Gd^4}{8n_c D^3} \quad \text{Equation 14}$$

where:  $G$  = modulus of rigidity (12x10<sup>6</sup> psi for steel)

**Example:** a 5" diameter coil compression spring, made with 0.7" diameter steel wire and has a total of 6 coils. Both ends are ground and flattened.

- Calculate the stiffness (answer:  $n_c = 4$ ,  $k = 720$  lb/in)
- Calculate the surge frequencies ( $f_{1,2,3} = 104, 208, 312$  Hz ...)
- Calculate the static deflection and system resonant frequency if 6 identical springs support a machine weighing 13000 lb. (3", 1.8Hz)

### 12.9 Two Level Model (2 DOF)

If the supporting floor is sufficiently flexible, then a two level (or higher order) model is a more accurate representation. This model is also useful for the case where an inertia block (seismic mass) is used between the machine and the rigid foundation.

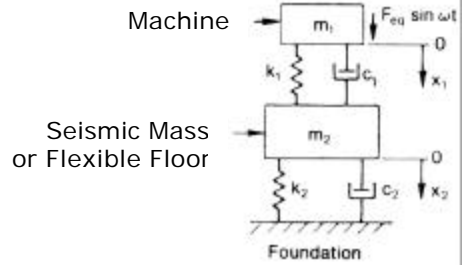


Figure 7. Two degree of freedom model of machinery isolation

If we neglect damping, the motions of the masses,  $m_1$  and  $m_2$  to an input force on the upper mass of  $F_{EQ} \sin \omega t$  are:

$$X_1 = \frac{(k_1 + k_2 - m_2 \omega^2) F_{EQ}}{(k_1 - m_1 \omega^2)(k_1 + k_2 - m_2 \omega^2) - k_1^2} \sin \omega t \tag{Equation 15}$$

$$X_2 = \frac{-k_1 F_{EQ}}{(k_1 - m_1 \omega^2)(k_1 + k_2 - m_2 \omega^2) - k_1^2} \sin \omega t \tag{Equation 16}$$

The transmissibility (ratio of force transmitted to foundation to the input force), is:

$$T = \left| \frac{F_T}{F_{EQ}} \right| = \left| \frac{k_2 X_2}{F_{EQ}} \right| = \left| \frac{-k_1 k_2}{(k_1 - m_1 \omega^2)(k_1 + k_2 - m_2 \omega^2) - k_1^2} \right| \tag{Equation 17}$$

The natural frequencies of the system are the two values of  $\omega$  where the denominator is equal to zero. In the old days, we would have factored this equation into the form  $a\omega^4 + b\omega^2 + c = 0$ , then

found the two roots using the quadratic equation  $\omega_{1,2}^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ . Now it's much more convenient to put equation 12 into a spreadsheet and calculate the transmissibility over a range of frequencies, as in Figure 9. There are now two natural frequencies, and two frequencies at which the system can resonate. The slope above the second resonance is now  $-40$  dB/decade

**Example:** An air conditioner chiller unit shown in Figure 8 is located directly above a suite of operating rooms in a hospital. Six air springs are used to isolate the chiller frame from a seismic mass (a block of reinforced cast concrete). Each spring has a stiffness of 73.5 lb/in (@ 68.5 psi). The seismic mass (2870 lbs) is isolated from the floor by six additional air springs, each with a stiffness of 736 lb/in (@ 50 psi). The chiller unit and frame weigh 3600 lbs. Determine the transmissibility at the lowest vibration frequency generated by the chiller, 29 Hz.

**Answer:**  $T = 2.6E-5$  @ 29 Hz, (46 dB of isolation, or 99.9974% effective), natural frequencies at 1.04 and 4.08 Hz, see Figure 9. Also shown in Figure 9 is the transmissibility if the upper springs and the seismic mass are eliminated (single degree of freedom system) and the system is supported on 3" static deflection springs ( $k = 1200$  lb/in). The resulting transmissibility at 29 Hz is now only .00389 (24 dB of isolation). The natural frequency is 1.8 Hz. As seen in this example, a two layer isolation system provides much greater isolation, at the added expense of additional springs, additional mass that must be supported by the floor, and one more resonant frequency with which to contend.

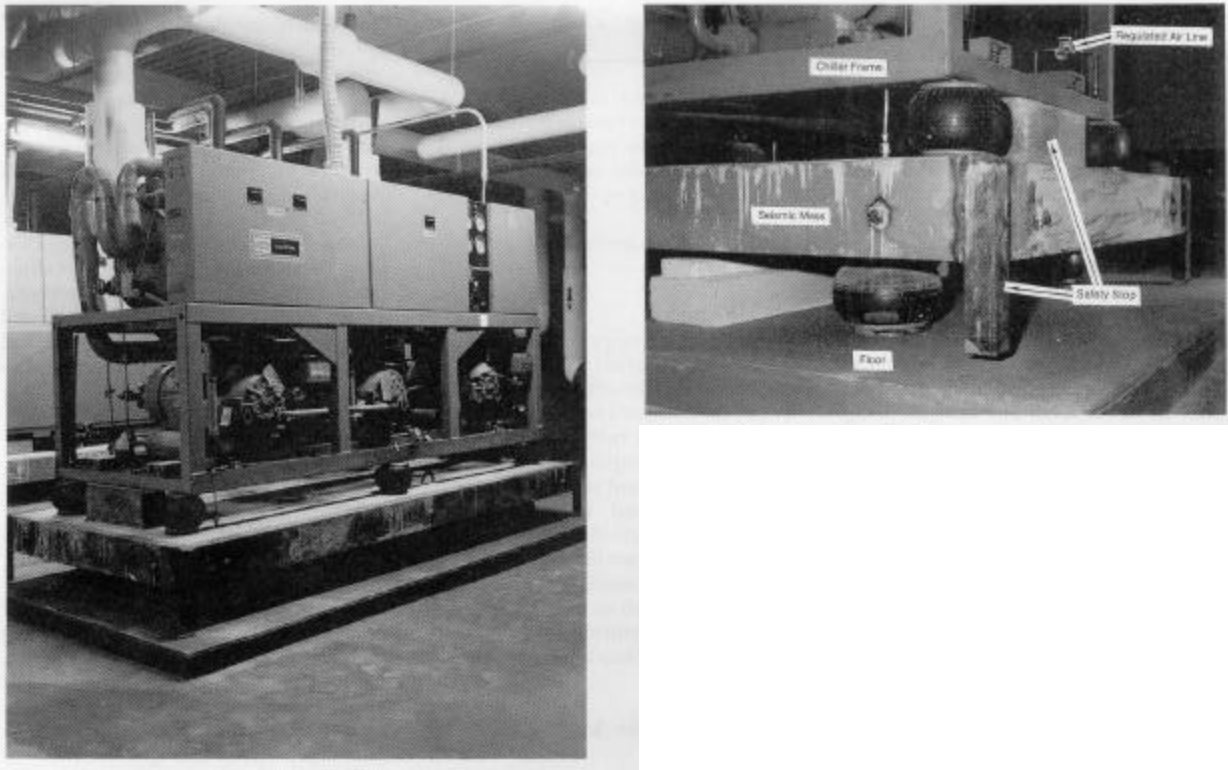


Figure 8. Air conditioner chiller installation, featuring a two level isolation system with a seismic mass (ref. Sound and Vibration, March 1985)

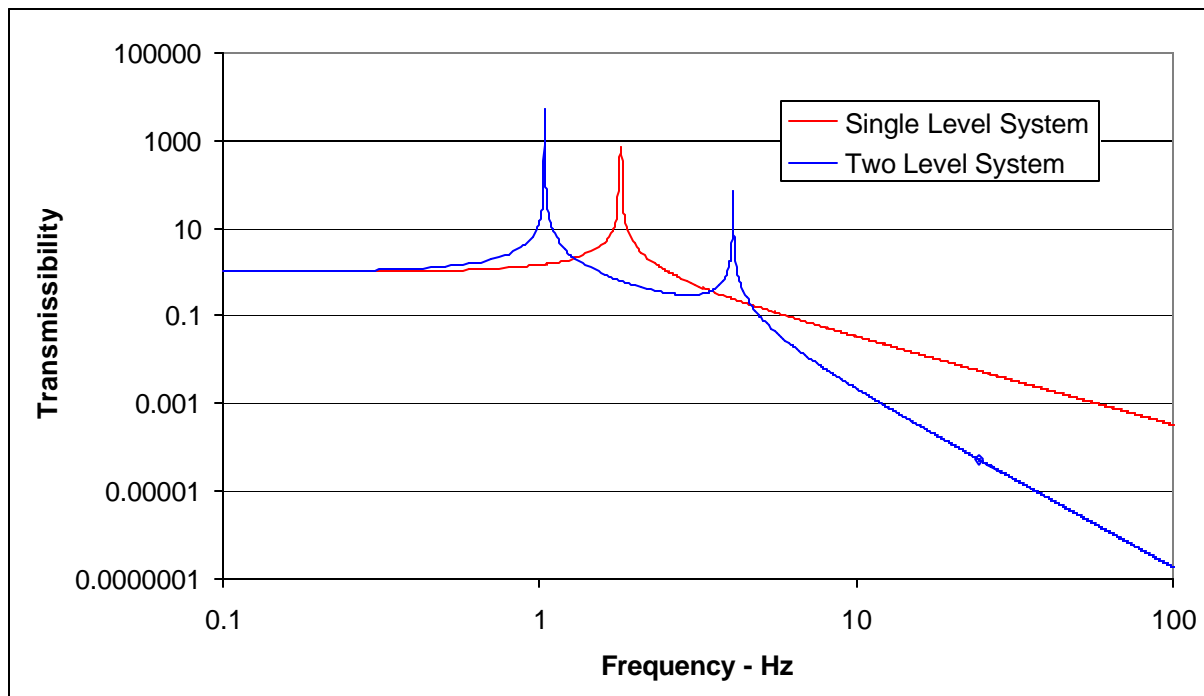


Figure 9. Transmissibility for air conditioner chiller example

- a) Two Level System:  $m_1 = 3600$  lb,  $m_2 = 2870$  lb,  $k_1 = 441$  lb/in,  $k_2 = 4416$  lb/in
- b) Single Level System:  $m = 3600$  lb,  $k = 1200$  lb/in

## 12.10 Higher Order Vibration Modes

The previous sections assumed a simple translational motion of the machine and its supports in the vertical direction. This is sometimes called the “bounce” mode. Higher order, rotational modes are also possible, including pitch and roll (Figure 10). These modes are not desirable and hopefully are of a lower magnitude than the bounce mode. Proper location of machinery on the isolators can minimize the excitation of these modes. Ideally the machine should be located on its base frame and isolators so that excitation forces act through the center of gravity of the system, so as not to excite these rotational modes. Large (wide) inertia blocks can also be used to control these modes.

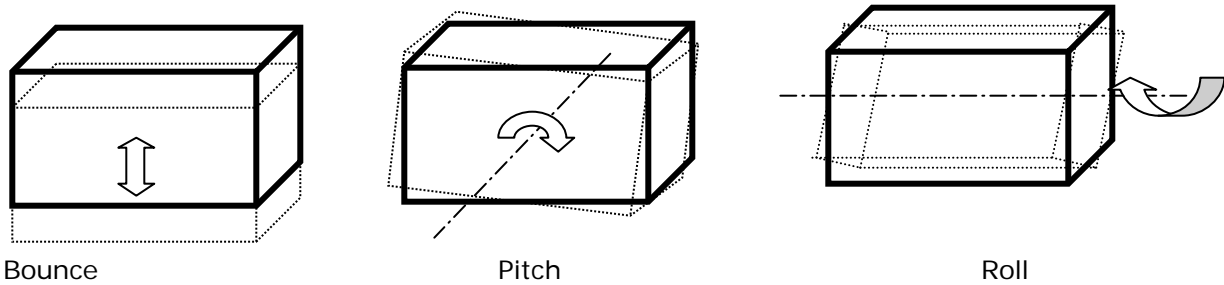


Figure 11. Rigid body modes of a machine on flexible supports